Organizing Your Foldables

Have students make this Foldable to help them organize and store their chapter Foldables. Begin with one sheet of 11" x 17" paper.

**Fold**
Fold the paper in half lengthwise. Then unfold.

**Fold and Glue**
Fold the paper in half widthwise and glue all of the edges.

**Glue and Label**
Glue the left, right, and bottom edges of the Foldable to the inside back cover of your Noteables notebook.

**Reading and Taking Notes** As you read and study each chapter, record notes in your chapter Foldable. Then store your chapter Foldables inside this Foldable organizer.
Contents

CHAPTER 1
Foldables ............................................. 1
Vocabulary Builder .................................. 2
1-1 A Plan for Problem Solving ...................... 4
1-2 Powers and Exponents ......................... 6
1-3 Squares and Square Roots .................... 8
1-4 Order of Operations ......................... 10
1-5 Problem-Solving Investigation:
   Guess and Check .................................. 12
1-6 Algebra: Variables and Expressions ........... 13
1-7 Algebra: Equations ............................ 15
1-8 Algebra: Properties ............................ 18
1-9 Algebra: Arithmetic Sequences .............. 20
1-10 Algebra: Equations and Functions .......... 23
Study Guide ........................................ 26

CHAPTER 2
Foldables ............................................. 31
Vocabulary Builder .................................. 32
2-1 Integers and Absolute Value .................... 34
2-2 Comparing and Ordering Integers .......... 36
2-3 The Coordinate Plane .......................... 38
2-4 Adding Integers ................................ 41
2-5 Subtracting Integers ........................... 44
2-6 Multiplying Integers ........................... 46
2-7 Problem-Solving Investigation:
   Look for a Pattern .............................. 48
2-8 Dividing Integers .............................. 49
Study Guide ........................................ 50

CHAPTER 3
Foldables ............................................. 54
Vocabulary Builder .................................. 55
3-1 Writing Expressions and Equations ......... 57
3-2 Solving Addition and Subtraction
   Equations ......................................... 59
3-3 Solving Multiplication Equations .......... 61
3-4 Problem-Solving Investigation:
   Work Backward .................................. 63
3-5 Solving Two-Step Equations ................. 64
3-6 Measurement: Perimeter and Area .......... 67
3-7 Functions and Graphs .......................... 69
Study Guide ........................................ 72

CHAPTER 4
Foldables ............................................. 76
Vocabulary Builder .................................. 77
4-1 Prime Factorization ............................ 79
4-2 Greatest Common Factor ..................... 81
4-3 Problem-Solving Investigation:
   Make an Organized List ......................... 84
4-4 Simplifying Fractions .......................... 85
4-5 Fractions and Decimals ....................... 87
4-6 Fractions and Percents ....................... 90
4-7 Percents and Decimals ....................... 92
4-8 Least Common Multiple ....................... 94
4-9 Comparing and Ordering Rational
   Numbers ........................................... 96
Study Guide ........................................ 98

CHAPTER 5
Foldables ............................................. 102
Vocabulary Builder .................................. 103
5-1 Estimating with Fractions ..................... 104
5-2 Adding and Subtracting Fractions .......... 106
5-3 Adding and Subtracting Mixed
   Numbers ........................................... 108
5-4 Problem-Solving Investigation:
   Estimate Possibilities ......................... 110
5-5 Multiplying Fractions and Mixed
   Numbers ........................................... 111
5-6 Algebra: Solving Equations ................. 113
5-7 Dividing Fractions and Mixed
   Numbers ........................................... 115
Study Guide ........................................ 117

CHAPTER 6
Foldables ............................................. 120
Vocabulary Builder .................................. 121
6-1 Ratios ........................................... 123
6-2 Rates ............................................. 125
6-3 Rate of Change and Slope ................. 128
6-4 Measurement: Changing Customary
   Units .............................................. 130
6-5 Measurement: Changing Metric
   Units .............................................. 133
6-6 Algebra: Solving Proportions ............. 136
6-7 Problem-Solving Investigation:
   Draw a Diagram ................................ 138
6-8 Scale Drawings ................................ 139
6-9 Fractions, Decimals, and Percents ....... 142
Study Guide ........................................ 144
## Contents

### CHAPTER 7

Foldables .................................................. 148  
Vocabulary Builder ................................. 149  
7-1 Percent of a Number ............... 151  
7-2 The Percent Proportion ........... 153  
7-3 Percent and Estimation .......... 155  
7-4 Algebra: The Percent Equation ... 157  
7-5 Problem-Solving Investigation: 
   Determine Reasonable Answers ..... 159  
7-6 Percent of Change ................. 160  
7-7 Sales Tax and Discount .......... 162  
7-8 Simple Interest ...................... 165  
Study Guide ........................................... 167

### CHAPTER 8

Foldables .................................................. 172  
Vocabulary Builder ................................. 173  
8-1 Line Plots ........................................... 175  
8-2 Measures of Central Tendency and 
   Range .............................................. 177  
8-3 Stem-and-Leaf Plots ................. 180  
8-4 Bar Graphs and Histograms ....... 184  
8-5 Problem-Solving Investigation: 
   Use a Graph ....................................... 188  
8-6 Using Graphs to Predict .......... 189  
8-7 Using Data to Predict ............. 191  
8-8 Using Sampling to Predict ...... 193  
8-9 Misleading Statistics ............ 195  
Study Guide ........................................... 197

### CHAPTER 9

Foldables .................................................. 201  
Vocabulary Builder ................................. 202  
9-1 Simple Events ......................... 204  
9-2 Sample Spaces ......................... 206  
9-3 The Fundamental Counting 
   Principle ......................................... 209  
9-4 Permutations ............................ 210  
9-5 Combinations ......................... 212  
9-6 Problem-Solving Investigation: 
   Act It Out ....................................... 214  
9-7 Theoretical and Experimental 
   Probability ....................................... 215  
9-8 Compound Events .................. 217  
Study Guide ........................................... 219

### CHAPTER 10

Foldables .................................................. 224  
Vocabulary Builder ................................. 225  
10-1 Angle Relationships ............. 227  
10-2 Complementary and Supplementary 
   Angles ........................................... 230  
10-3 Statistics: Display Data in a Circle 
   Graph ........................................... 232  
10-4 Triangles ......................... 236  
10-5 Problem-Solving Investigation: 
   Use Logical Reasoning ........... 238  
10-6 Quadrilaterals ................. 239  
10-7 Similar Figures .................... 241  
10-8 Polygons and Tessellations .... 243  
10-9 Translations ..................... 245  
10-10 Reflections ....................... 247  
Study Guide ........................................... 250

### CHAPTER 11

Foldables .................................................. 254  
Vocabulary Builder ................................. 255  
11-1 Area of Parallelograms ........... 257  
11-2 Area of Triangles and Trapezoids . 258  
11-3 Circles and Circumference ....... 260  
11-4 Area of Circles ................. 262  
11-5 Problem-Solving Investigation: 
   Solve a Simpler Problem .......... 263  
11-6 Area of Complex Figures ....... 264  
11-7 Three-Dimensional Figures .... 266  
11-8 Drawing Three-Dimensional 
   Figures .......................................... 268  
11-9 Volume of Prisms ............ 270  
11-10 Volume of Cylinders .......... 271  
Study Guide ........................................... 273

### CHAPTER 12

Foldables .................................................. 278  
Vocabulary Builder ................................. 279  
12-1 Estimating Square Roots ...... 280  
12-2 The Pythagorean Theorem ...... 282  
12-3 Problem-Solving Investigation: 
   Make a Model ......................... 285  
12-4 Surface Area of Rectangular Prisms . 286  
12-5 Surface Area of Cylinders ...... 289  
Study Guide ........................................... 291
EXAMPLE

There are 76 thousand acres of state parkland in Georgia. This is 4 thousand acres more than three times the number of acres of state parkland in Mississippi. How many acres of state parkland are there in Mississippi?

Three times the number of acres of state parkland in Mississippi plus 4,000 is 76,000.

Let \( m \) = the acres of state parkland in Mississippi.

Three times the number of acres of parkland in Mississippi plus 4,000 is 76,000.

\[ 3m + 4,000 = 76,000 \]

Write the equation.

Subtract 4,000 from each side.

\[ 3m = 72,000 \]

Simplify.

Divide each side by 3.

\[ m = 24,000 \]

Simplify.

There are 24,000 acres of state parkland in Mississippi.

Check Your Progress

BASEBALL: Matthew had 64 hits during last year’s baseball season. This was 8 less than twice the number of hits Gregory had. How many hits did Gregory have during last year’s baseball season?

The examples can also be found on the Interactive Classroom CD-ROM.

The examples correspond to the In-Class Examples in the margin of the Teacher Edition.

The Check Your Progress Exercises are also found on the Interactive Classroom CD-ROM.
This note-taking guide is designed to help students succeed in *Math Connects, Course 2*. Each chapter includes:

**Chapter Opener**
Contains instructions and illustrations for students to make a Foldable that will help them organize their notes.

**Foldable**
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

1. **Begin with a sheet of 11" x 17" paper.**
2. **Fold the short sides toward the middle.**
3. **Fold the top to the bottom.**
4. **Open. Cut along the second fold to make four tabs.**
5. **Label each of the tabs as shown.**

**Note-Taking Tip:**
When you take notes, listen or read for main ideas. Then record those ideas in a simplified form for future reference.

**Build Your Vocabulary**
This is an alphabetical list of new vocabulary terms you will learn in Chapter 3. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition Property of Equality</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Division Property of Equality</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>formula</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>linear equation</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**A Note-Taking Tip**
Provides a helpful hint students can use when taking notes.

**The Build Your Vocabulary table** allows students to write definitions and examples of important vocabulary terms together in one convenient place.

**Within each chapter, Build Your Vocabulary boxes** will remind students to fill in this table.
Solve an Addition Equation

14 + y = 20

Check
Write the equation.

Solve each equation. Check your solution.

7. 14 + y = 20

8. 35 - z = 15

Foldables feature
reminds students to
take notes in their
Foldable.

Lessons cover the content of
the lessons in the textbook.
As you discuss each example,
have students follow along and
complete the fill-in boxes. Be
sure to remind them to take
notes as appropriate.

Examples parallel the
examples in the textbook.

Math Connects, Course 2

Bringing It All Together

STUDY GUIDE

VOCABULARY

BUILD YOUR VOCABULARY

Homework Assignment

72

Math Connects, Course 2

Bringing It All Together
Study Guide reviews
the main ideas and key
concepts from each lesson.
**NOTE-TAKING TIPS**

Notes are a reminder to the students as to what they learned in class. Taking good notes can help students succeed in mathematics. The following tips will help students take better classroom notes.

- **Before class,** ask what your teacher will be discussing in class. Review mentally what you already know about the concept.

- **Be an active listener.** Focus on what your teacher is saying. Listen for important concepts. Pay attention to words, examples, and/or diagrams your teacher emphasizes.

- **Write your notes as clear and concise as possible.** The following symbols and abbreviations may be helpful in your note-taking.

<table>
<thead>
<tr>
<th>Word or Phrase</th>
<th>Symbol or Abbreviation</th>
<th>Word or Phrase</th>
<th>Symbol or Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>for example</td>
<td>e.g.</td>
<td>not equal</td>
<td>≠</td>
</tr>
<tr>
<td>such as</td>
<td>i.e.</td>
<td>approximately</td>
<td>≈</td>
</tr>
<tr>
<td>with</td>
<td>w/</td>
<td>therefore</td>
<td>∴</td>
</tr>
<tr>
<td>without</td>
<td>w/o</td>
<td>versus</td>
<td>vs</td>
</tr>
<tr>
<td>and</td>
<td>+</td>
<td>angle</td>
<td>∡</td>
</tr>
</tbody>
</table>

- **Use a symbol** such as a star (★) or an asterisk (*) to emphasize important concepts. Place a question mark (?) next to anything that you do not understand.

- **Ask questions** and participate in class discussion.

- **Draw and label** pictures or diagrams to help clarify a concept.

- When working out an example, write what you are doing to solve the problem next to each step. Be sure to use your own words.

- **Review your notes** as soon as possible after class. During this time, organize and summarize new concepts and clarify misunderstandings.

**Note-Taking Don’ts**

- **Don’t** write every word. Concentrate on the main ideas and concepts.

- **Don’t** use someone else’s notes as they may not make sense.

- **Don’t** doodle. It distracts you from listening actively.

- **Don’t** lose focus or you will become lost in your note-taking.
Introduction to Algebra and Functions

Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin this Interactive Study Notebook to help you in taking notes.

Begin with eleven sheets of notebook paper.

**STEP 1** *Staple* the eleven sheets together to form a booklet.

**STEP 2** *Make* each one 2 lines longer than the one before it.

**STEP 3** *Write* the chapter title on the cover and label each tab with the lesson number.

**NOTE-TAKING TIP:** When taking notes, it is often a good idea to write a summary of the lesson in your own words. Be sure to paraphrase key points.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 1. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term's definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>algebra</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>algebraic expression</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[al-juh-BRAY-ihk]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>arithmetic sequence</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[air-ith-MEH-tik]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>base</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>coefficient</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>defining the variable</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>domain</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>equation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ih-KWAY-zhuhn]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>equivalent expression</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>evaluate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>exponent</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition</td>
<td>Description or Example</td>
</tr>
<tr>
<td>---------------------</td>
<td>---------------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>factors</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>function</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>function rule</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>numerical expression</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>order of operations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>perfect square</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>powers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>radical sign</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>range</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sequence</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>solution</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>square</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>square root</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>square root</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>term</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>variable</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
EXAMPLE Use the Four-Step Plan

**SPENDING** A can of soda holds 12 fluid ounces. A 2-liter bottle holds about 67 fluid ounces. If a pack of six cans costs the same as a 2-liter bottle, which is the better buy?

**UNDERSTAND** What are you trying to find? You know the number of fluid ounces of soda in one can of soda. You need to know the number of fluid ounces of soda in a pack of six cans.

**PLAN** You can find the number of fluid ounces of soda in a pack of six cans by **multiplying** the number of fluid ounces in one can by **six**.

**SOLVE**

\[ 12 \times 6 = 72 \]

There are **72** fluid ounces of soda in a pack of six cans. The number of fluid ounces of soda in a 2-liter bottle is about **67**. Therefore, the **pack of six cans** is the better buy because you get more soda for the same price.

**CHECK** The answer makes sense based on the facts given in the problem.

**Check Your Progress** **FIELD TRIP** The sixth-grade class at Meadow Middle School is taking a field trip to the local zoo. There will be 142 students plus 12 adults going on the trip. If each school bus can hold 48 people, how many buses will be needed for the field trip?

4 buses
EXAMPLE

Use a Strategy in the Four-Step Plan

POPULATION For every 100,000 people in the United States, there are 5,750 radios. For every 100,000 people in Canada, there are 323 radios. Suppose Sheamus lives in Des Moines, Iowa, and Alex lives in Windsor, Ontario. Both cities have about 200,000 residents. About how many more radios are there in Sheamus’s city than in Alex’s city?

UNDERSTAND You know the approximate number of radios per 100,000 people in both Sheamus’s city and Alex’s city.

PLAN You can find the approximate number of radios in each city by multiplying the estimate per 100,000 people by two to get an estimate per 200,000 people. Then, subtract to find how many more radios there are in Des Moines than in Windsor.

SOLVE Des Moines: \(5,750 \times 2 = 11,500\)

Windsor: \(323 \times 2 = 646\)

\(11,500 - 646 = 10,854\)

So, Des Moines has about \(10,854\) more radios than Windsor.

CHECK Based on the information given in the problem, the answer seems to be reasonable.

KEY CONCEPTS

Problem-Solving Strategies
- guess and check
- look for a pattern
- make an organized list
- draw a diagram
- act it out
- solve a simpler problem
- use a graph
- work backward
- eliminate possibilities
- estimate reasonable answers
- use logical reasoning
- make a model

READING Ben borrows a 500-page book from the library. On the first day, he reads 24 pages. On the second day, he reads 39 pages and on the third day he reads 54 pages. If Ben follows the same pattern of number of pages read for seven days, will he have finished the book at the end of the week?

No; he will have only read 483 pages.
Main Idea

- Use powers and exponents.

Build Your Vocabulary (pages 2–3)

Two or more numbers that are multiplied together to form a **product** are called **factors**.

The **exponent** tells how many times the base is used as a **factor**.

The **base** is the common **factor**.

Numbers expressed using **exponents** are called **powers**.

Five to the **second** power is five **squared**.

Four to the **third** power is four **cubed**.

Examples: Write Powers as Products

Write each power as a product of the same factor.

1. **$8^4$**
   - Eight is used as a factor **four** times. $8^4 = 8 \cdot 8 \cdot 8 \cdot 8$

2. **$4^6$**
   - Four is used as a factor six times. $4^6 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

Check Your Progress: Write each power as a product of the same factor.

a. **$3^6$**
   - $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

b. **$7^3$**
   - $7 \cdot 7 \cdot 7$
You can evaluate, or find the value of, powers by multiplying the factors.

Numbers written without exponents are in standard form.

Numbers written with exponents are in exponential form.

**EXAMPLES** Write Powers in Standard Form

Evaluate each expression.

1. \(8^3 = 8 \cdot 8 \cdot 8 = 512\)
2. \(6^4 = 6 \cdot 6 \cdot 6 \cdot 6 = 1,296\)

**Check Your Progress** Evaluate each expression.

a. \(4^4 = 256\)

b. \(5^5 = 3,125\)

**EXAMPLE** Write Numbers in Exponential Form

Write \(9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9\) in exponential form.

9 is the base. It is used as a factor 6 times.

So, the exponent is 6.

\(9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 \cdot 9 = 9^6\)

**Check Your Progress** Write \(3 \cdot 3 \cdot 3 \cdot 3 \cdot 3\) in exponential form.

\(3^5\)

**HOMEWORK ASSIGNMENT**

Page(s): 
Exercises: 
Squares and Square Roots

**Main Idea**
*Find squares of numbers and square roots of perfect squares.*

**Build Your Vocabulary** (pages 2–3)

- The **product** of a number and **itself** is the **square** of the number.
- **Perfect squares** like 9, 16, and 225 are squares of whole numbers.
- The **factors** multiplied to form perfect squares are called **square roots**.
- A **radical sign**, √, is the symbol used to indicate the positive **square root** of a number.

**Examples**

Find the square of 5.

Multiply 5 by **itself**.

\[ 5 \times 5 = 25 \]

**METHOD 1**

Use paper and pencil.

\[ 19 \times 19 = 361 \]

**METHOD 2**

Use a calculator.

\[ 19 \times 19 \text{ ENTER} = 361 \]

**Check Your Progress**

Find the square of each number.

- a. 7
  
  49

- b. 21
  
  441
Find Square Roots

Find $\sqrt{36}$.

What number times itself is 36?

$6 \cdot 6 = 36$, so $\sqrt{36} = 6$.

Find $\sqrt{676}$.

Use a calculator.

$$\text{2nd} \ x^2 \ 676 \ \text{ENTER} \ 26$$

So, $\sqrt{676} = 26$.

Check Your Progress

Find each square root.

a. $\sqrt{64}$

b. $\sqrt{529}$

8

23

GAMES A checkerboard is a square with an area of 1,225 square centimeters. What are the dimensions of the checkerboard?

The checkerboard is a square. By finding the square root of the area, 1,225, you find the length of one side.

$$\text{2nd} \ x^2 \ 1225 \ \text{ENTER} \ 35$$

Use a calculator.

The dimensions of the checkerboard are $35$ cm by $35$ cm.

Check Your Progress GARDENING Kyle is planting a new garden that is a square with an area of 42.25 square feet. What are the dimensions of Kyle’s garden?

$$\text{2nd} \ x^2 \ 42.25 \ \text{ENTER} \ 6.5$$

The dimensions of Kyle’s garden are 6.5 feet by 6.5 feet.
Build Your Vocabulary (pages 2–3)

The expressions $4 \cdot 6 - (5 + 7)$ and $8 \cdot (9 - 3) + 4$ are numerical expressions.

Order of operations are rules that ensure that numerical expressions have only one value.

Main Idea

• Evaluate expressions using the order of operations.

Key Concept

Order of Operations

1. Evaluate the expressions inside grouping symbols.
2. Evaluate all powers.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

Foldables Be sure to include the order of operations on the Lesson 1-4 page of your Foldable.

Examples Evaluate Expressions

Evaluate each expression.

1. $27 - (18 + 2)$

$27 - (18 + 2) = 27 - 20$ Add first since $18 + 2$ is in parentheses.

\[= 7\] Subtract 20 from 27.

2. $15 + 5 \cdot 3 - 2$

$15 + 5 \cdot 3 - 2 = 15 + 15 - 2$ Multiply 5 and 3.

\[= 30 - 2\] Add 15 and 15.

\[= 28\] Subtract 2 from 30.

Check Your Progress Evaluate each expression.

a. $45 - (26 + 3)$

\[16\]

b. $32 - 3 \cdot 7 + 4$

\[15\]
EXAMPLES  Use Order of Operations

Evaluate each expression.

3  \(12 \times 3 - 2^2\)

\[
12 \times 3 - 2^2 = 12 \times 3 - 4
\]

Find the value of \(2^2\).

\[
= 36 - 4
\]

Multiply 12 and 3.

\[
= 32
\]

Subtract 4 from 36.

4  \(28 \div (3 - 1)^2\)

\[
28 \div (3 - 1)^2 = 28 \div 2^2
\]

Subtract 1 from 3 inside the parentheses.

\[
= 28 \div 4
\]

Find the value of \(2^2\).

\[
= 7
\]

Divide.

Check Your Progress  Evaluate each expression.

a. \(9 \times 5 + 3^2\)  
   \[
   9 \times 5 + 3^2 = 9 \times 5 + 9
   \]
   \[
   = 45 + 9
   \]
   \[
   = 54
   \]
   b. \(36 \div (14 - 11)^2\)  
   \[
   36 \div (14 - 11)^2 = 36 \div 3^2
   \]
   \[
   = 36 \div 9
   \]
   \[
   = 4
   \]

EXAMPLE  Evaluate an Expression

MONEY Julian is buying one box of favors, one box of balloons, and three rolls of crepe paper. What is the total cost?

<table>
<thead>
<tr>
<th>Item</th>
<th>Quantity</th>
<th>Unit Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>crepe paper</td>
<td>3 rolls</td>
<td>$2</td>
</tr>
<tr>
<td>favors</td>
<td>1 box</td>
<td>$7</td>
</tr>
<tr>
<td>balloons</td>
<td>1 box</td>
<td>$5</td>
</tr>
</tbody>
</table>

\[
1 \times 7 + 1 \times 5 + 3 \times 2 = 7 + 5 + 6 \text{ or } 18
\]

The total cost is \(\$18\).

Check Your Progress  What is the total cost of two boxes of favors, two boxes of balloons, and six rolls of crepe paper?

\(\$36\)
CONCESSIONS The concession stand at the school play sold lemonade for $0.50 and cookies for $0.25. They sold 7 more lemonades than cookies, and they made a total of $39.50. How many lemonades and cookies were sold?

UNDERSTAND You know the cost of each lemonade and cookie. You know the total amount made and that they sold 7 more lemonades than cookies. You need to know how many lemonades and cookies were sold.

PLAN Make a guess and check it. Adjust the guess until you get the correct answer.

SOLVE Make a guess.

14 cookies, 21 lemonades \(0.25(14) + 0.50(21)\)

This guess is too low \(= \$14.00\)

50 cookies, 57 lemonades \(0.25(50) + 0.50(57)\)

This guess is too high \(= \$41.00\)

48 cookies, 55 lemonades \(0.25(48) + 0.50(55)\)

\(= \$39.50\)

CHECK 48 cookies cost $12 and 55 lemonades cost $27.50. Since $12 + $27.50 = $39.50 and 55 is 7 more than 48, the guess is correct.

Check Your Progress ZOO A total of 122 adults and children went to the zoo. Adult tickets cost $6.50 and children’s tickets cost $3.75. If the total cost of the tickets was $597.75, how many adults and children went to the zoo?

51 adults and 71 children
**Main Idea**

- Evaluate simple algebraic expressions.

**Build Your Vocabulary** (pages 2–3)

You can use a **placeholder**, or **variable**, in an expression.

The expression $7 + n$ is called an **algebraic** expression.

The branch of mathematics that involves expressions with **variables** is called **algebra**.

The **numerical** factor of a term that contains a variable is called a **coefficient**.

**Examples** Evaluate Expressions

1. Evaluate $t - 4$ if $t = 6$.

   
   \[
   t - 4 = 6 - 4 \\
   = 2
   \]

   Replace $t$ with $6$. Subtract.

2. Evaluate $5x + 3y$ if $x = 7$ and $y = 9$.

   
   \[
   5x + 3y = 5 \cdot 7 + 3 \cdot 9 \\
   = 35 + 27 \\
   = 62
   \]

   Replace $x$ with $7$ and $y$ with $9$. Do all multiplications first. Add $35$ and $27$.

3. Evaluate $5 + a^2$ if $a = 5$.

   
   \[
   5 + a^2 = 5 + 5^2 \\
   = 5 + 25 \\
   = 30
   \]

   Replace $a$ with $5$. Evaluate the power. Add.
Check Your Progress  Evaluate each expression.

a. \( 7 + m \) if \( m = 4 \).

\[
11
\]

b. \( 4a - 2b \) if \( a = 9 \) and \( b = 6 \).

\[
24
\]

c. \( 24 - s^2 \) if \( s = 3 \).

\[
15
\]

EXAMPLE  Evaluate an Expression

**TEMPERATURE** The formula for rewriting a Fahrenheit temperature as a Celsius temperature is \( \frac{5(F - 32)}{9} \), where \( F \) equals the temperature in degrees Fahrenheit. Find the Celsius equivalent of \( 99^\circ F \).

\[
\frac{5(F - 32)}{9} = \frac{5(99 - 32)}{9}
\]

Replace \( F \) with 99.

\[
= \frac{5(67)}{9} = \frac{335}{9}
\]

Subtract \( 32 \) from 99 and multiply.

\[
\approx 37.2
\]

Divide 335 by 9.

The Celsius equivalent of \( 99^\circ F \) is about \( 37.2^\circ C \).

**Check Your Progress** **BOWLING** David’s cost for bowling can be described by the formula \( 1.75 + 2.5g \), where \( g \) is the number of games David bowls. Find the total cost of bowling if David bowls 3 games.

\[
\$9.25
\]
**Main Idea**

- Write and solve equations using mental math.

**Build Your Vocabulary** (pages 2–3)

An equation is a **sentence** in mathematics that contains an equals sign.

The **solution** of an equation is a number that makes the sentence **true**.

The process of finding a **solution** is called **solving an equation**.

When you choose a **variable** to represent one of the unknowns in an equation, you are **defining the variable**.

---

**Example: Solve an Equation Mentally**

1. Solve \( p - 14 = 5 \) mentally.

   \[
   p - 14 = 5 \quad \text{Write the equation.}
   \]

   \[
   19 - 14 = 5 \quad \text{You know that } 19 - 14 \text{ is } 5.
   \]

   \[
   5 = 5 \quad \text{Simplify.}
   \]

   The solution is **19**.

**Check Your Progress**

Solve \( p - 6 = 11 \) mentally.

17
A store sells pumpkins for $2 per pound. Paul has $18. Use the equation \(2x = 18\) to find how large a pumpkin Paul can buy with $18.

A 6 lb  
B 7 lb  
C 8 lb  
D 9 lb

Read the Item

Solve \(2x = 18\) to find how many pounds the pumpkin can weigh.

Solve the Item

\[
2x = 18
\]

Write the equation.

\[
2 \cdot 9 = 18
\]

You know that \(2 \cdot 9\) is 18.

Paul can buy a pumpkin as large as 9 pounds.

The answer is D.

Check Your Progress

A store sells notebooks for $3 each. Stephanie has $15. Use the equation \(3x = 15\) to find how many notebooks she can buy with $15.

F 4  
G 5  
H 6  
J 7

G
**EXERCISE 5**

**ENTERTAINMENT** An adult paid $18.50 for herself and two students to see a movie. If the two student tickets cost $11 together, what is the cost of the adult ticket?

**Words**
The cost of one adult ticket and two student tickets is $18.50.

**Variable**
Let \(a\) represent the cost of an adult movie ticket.

**Equation**
\[a + 11 = 18.50\]

Write the equation.

\[\begin{align*}
7.50 + 11 &= 18.50 \\
18.50 &= 18.50
\end{align*}\]

Replace \(a\) with 7.50 to make the equation true.

Simplify.

The number **7.50** is the solution of the equation. So, the cost of an adult movie ticket is **$7.50**.

**Check Your Progress** **ICE CREAM** Julie spends $9.50 at the ice cream parlor. She buys a hot fudge sundae for herself and ice cream cones for each of the three friends who are with her. Find the cost of Julie’s sundae if the three ice cream cones together cost $6.30.

**$3.20**
Main Idea
- Use Commutative, Associative, Identity, and Distributive properties to solve problems.

Organize It
On the Lesson 1-8 page your Foldable, be sure to include examples showing the addition and multiplication properties.

Main Idea
The expressions $5(9 + 2)$ and $5(9) + 5(2)$ are equivalent expressions because they have the same value.

Build Your Vocabulary (pages 2–3)

Examples
Use the Distributive Property

Use the Distributive Property to rewrite each expression. Then evaluate it.

1. $8(5 + 7)$
   
   $8(5 + 7) = 8 \cdot 5 + 8 \cdot 7$
   
   $= 40 + 56$  Multiply.
   
   $= 96$  Add.

2. $6(9) + 6(2)$
   
   $6(9) + 6(2) = 54 + 12$  Multiply.
   
   $= 66$  Add.

Check Your Progress
Use the Distributive Property to evaluate each expression.

a. $4(6 + 3)$

   $4(6) + 4(3); 36$

b. $(5 + 3)7$

   $5(7) + 3(7); 56$
**EXAMPLE**

**VACATIONS** Mr. Harmon has budgeted $150 per day for his hotel and meals during his vacation. If he plans to spend six days on vacation, how much will he spend?

\[ 6(150) = 6(100 + 50) \]
\[ = 6 \cdot 100 + 6 \cdot 50 \quad \text{Distributive Property} \]
\[ = 600 + 300 \quad \text{or} \quad 900 \quad \text{Multiply, then add.} \]

Mr. Harmon will spend about $900 on a six-day vacation.

**Check Your Progress**

**COOKIES** Heidi sold cookies for $2.50 per box for a fundraiser. If she sold 60 boxes of cookies, how much money did she raise?

\[ 60 \cdot 2.50 = 150 \]

**Build Your Vocabulary** (pages 2–3)

Properties are statements that are true for all numbers.

**EXAMPLE**

**Identify Properties**

Find \( 5 \cdot 13 \cdot 20 \) mentally. Justify each step.

\[ 5 \cdot 13 \cdot 20 = 5 \cdot 20 \cdot 13 \quad \text{Communtative Property of Multiplication} \]
\[ = (5 \cdot 20) \cdot 13 \quad \text{Associative Property of Multiplication} \]
\[ = 100 \cdot 13 \quad \text{or} \quad 1,300 \quad \text{Multiply 100 and 13 Mentally.} \]

**Check Your Progress** Name the property shown by the statement \( 4 + (6 + 2) = (4 + 6) + 2 \).

Associative Property of Addition
A sequence is an ordered list of numbers. Each number in a sequence is called a term. In an arithmetic sequence, each term is found by adding the same number to the previous term.

Describe Patterns in Sequences

Describe the relationship between the terms in each arithmetic sequence. Then write the next three terms in the sequence.

7, 11, 15, 19, ...

7, 11, 15, 19, ...

+ 4 + 4 + 4

Each term is found by adding 4 to the previous term.

Continue the pattern to find the next three terms.

19 + 4 = 23  23 + 4 = 27  27 + 4 = 31

The next three terms are 23, 27, and 31.
Each term is found by adding $0.4$ to the previous term.

Continue the pattern to find the next three terms.

\[
\begin{align*}
1.3 + 0.4 &= 1.7 \\
1.7 + 0.4 &= 2.1 \\
2.1 + 0.4 &= 2.5
\end{align*}
\]

The next three terms are 1.7, 2.1, and 2.5.

**Check Your Progress** Describe the relationship between the terms in each arithmetic sequence. Then write the next three terms in the sequence.

**a.** 13, 24, 35, 46, ...

Add 11; 57, 68, 79

**b.** 0.6, 1.5, 2.4, 3.3, ...

Add 0.9; 4.2, 5.1, 6.0

---

**WRITE IT**

In your own words, explain how to determine the pattern in a sequence.

---

**EXAMPLE Use a Table**

**EXERCISE** Mehmet started a new exercise routine. The first day, he did 2 sit-ups. Each day after that, he did 2 more sit-ups than the previous day. If he continues this pattern, how many sit-ups will he do on the tenth day?

<table>
<thead>
<tr>
<th>Position</th>
<th>Operation</th>
<th>Value of Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1 \cdot 2$</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>$2 \cdot 2$</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>$3 \cdot 2$</td>
<td>6</td>
</tr>
<tr>
<td>$d$</td>
<td>$d \cdot 2$</td>
<td>$2d$</td>
</tr>
</tbody>
</table>

(continued on the next page)
Each term is 2 times its position number. So, the expression is \(2n\).

\[2n\]

Write the expression.

\[2(10) = 20\]

Replace \(n\) with 10.

So, on the tenth day, Mehmet will do 20 sit-ups.

**Check Your Progress**

**CONCERTS** The first row of a theater has 8 seats. Each additional row has eight more seats than the previous row. If this pattern continues, what algebraic expression can be used to find the number of seats in the 15th row? How many seats will be in the 15th row?

8\(n\); 120 seats
Algebra: Equations and Functions

**Main Idea**
- Make function tables and write equations.

**Remember It**
When \(x\) and \(y\) are used in an equation, \(x\) usually represents the input and \(y\) usually represents the output.

**Build Your Vocabulary** (pages 2–3)
A relationship where one thing depends on another is called a **function**.

The **operation** performed on the input is given by the **function rule**.

You can organize the **input** numbers, **output** numbers, and the function rule in a **function table**.

The set of **input** values is called the **domain**.
The set of **output** values is called the **range**.

**Example** Make a Function Table
Asha earns $6.00 an hour working at a grocery store. Make a function table that shows Asha’s total earnings for working 1, 2, 3, and 4 hours.

<table>
<thead>
<tr>
<th>Input</th>
<th>Function</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Hours</td>
<td>Multiply by 6</td>
<td>Total Earnings ($)</td>
</tr>
<tr>
<td>1</td>
<td>(6 \times 1)</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>(6 \times 2)</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>(6 \times 3)</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>(6 \times 4)</td>
<td>24</td>
</tr>
</tbody>
</table>
Check Your Progress  MOVIE RENTAL  Dave goes to the video store to rent a movie. The cost per movie is $3.50. Make a function table that shows the amount Dave would pay for renting 1, 2, 3, and 4 movies.

<table>
<thead>
<tr>
<th>Input</th>
<th>Function Rule</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Movies</td>
<td>Multiply by 3.50</td>
<td>Total Cost ($)</td>
</tr>
<tr>
<td>1</td>
<td>3.50 × 1</td>
<td>3.50</td>
</tr>
<tr>
<td>2</td>
<td>3.50 × 2</td>
<td>7.00</td>
</tr>
<tr>
<td>3</td>
<td>3.50 × 3</td>
<td>10.50</td>
</tr>
<tr>
<td>4</td>
<td>3.50 × 4</td>
<td>14.00</td>
</tr>
</tbody>
</table>

**EXAMPLES**

**READING**  Melanie read 14 pages of a detective novel each hour. Write an equation using two variables to show how many pages \( p \) she read in \( h \) hours.

<table>
<thead>
<tr>
<th>Input</th>
<th>Function</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Hours ( (h) )</td>
<td>Multiply by 14</td>
<td>Number of Pages Read ( (p) )</td>
</tr>
<tr>
<td>1</td>
<td>( 1 \times 14 )</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>( 2 \times 14 )</td>
<td>28</td>
</tr>
<tr>
<td>3</td>
<td>( 3 \times 14 )</td>
<td>42</td>
</tr>
<tr>
<td>( h )</td>
<td>( 14h )</td>
<td>14( h )</td>
</tr>
</tbody>
</table>

**Words**  number of pages equals 14 pages times number of hours

**Variable**  Let \( p \) represent the number of pages read.

**Equation**  \( p = 14h \)
READING  Use your equation from Example 2 to find how many pages Melanie read in 7 hours.

\[ p = 14h \]

Write the equation.

\[ p = 14 \times 7 \]

Replace \( h \) with 7.

\[ p = 98 \]

Multiply.

Melanie read 98 pages in 7 hours.

Check Your Progress

a. TRAVEL  Derrick drove 55 miles per hour to visit his grandmother. Write an equation using two variables to show how many miles \( m \) he drove in \( h \) hours.

\[ m = 55h \]

b. TRAVEL  Use your equation from above to find how many miles Derrick drove in 6 hours.

330 miles
1-1 A Plan for Problem Solving

Underline the correct term to complete each sentence.

1. The (Plan, Solve) step is the step of the four-step plan in which you decide which strategy you will use to solve the problem.

2. According to the four-step plan, if your answer is not correct, you should (estimate the answer, make a new plan and start again).

3. Once you solve a problem, make sure your solution contains any appropriate (strategies, units or labels).

1-2 Powers and Exponents

Identify the exponent in each expression.

4. $5^8$ 8 5. $8^3$ 3

Evaluate each expression.

6. $4^3$ 64 7. $8^5$ 32,768

Complete the sentence.

8. Numbers written with exponents are in exponential form, whereas numbers written without exponents are in standard form.
Squares and Square Roots

Complete each sentence.

9. The square of 3 means \(3 \times 3\).

10. Nine units squared means 9 squares with sides of 1 unit each.

Find the square of each number.

11. 16 \(256\) 12. 28 \(784\)

Find the square root of each number.

13. \(\sqrt{121}\) 11 14. \(\sqrt{484}\) 22

Order of Operations

Evaluate each expression.

15. \(9 + 18 \div 6\) 16. \((7 - 4)^2 \div 3\)

17. \(2 \times 4^2 \div 4 - 1\) 18. \(8 + 2(9 - 5) - (2 \cdot 3)\)

Problem-Solving Investigation: Guess and Check

Solve using the guess and check strategy.

19. MONEY Gary deposited $38 into his savings account every week for eight weeks. At the end of this time, the total amount in his account was $729. How much money did Gary have in his account before the deposits?

$425
Algebra: Variables and Expressions

Evaluate each expression if \( a = 5 \) and \( b = 6 \).

20. \( 2a + 3b \)  
21. \( \frac{ab}{5} \)  
22. \( a^2 - 3b \)

Algebra: Equations

Solve each equation mentally.

23. \( 5 + b = 12 \)  
24. \( h - 6 = 3 \)  
25. \( 12 \cdot 4 = n \)  
26. \( 2 = \frac{x}{4} \)  
27. \( 9t = 54 \)  
28. \( 35 \div c = 7 \)

Algebra: Properties

Match the statement with the property it shows.

29. \( 5 + (3 + 6) = (5 + 3) + 6 \)  
30. \( 8 + 0 = 8 \)  
31. \( 4(7 - 2) = 4(7) - 4(2) \)  
32. \( 10 + 9 = 9 + 10 \)
Algebra: Arithmetic Sequences

Complete the sentence.

33. In an arithmetic sequence, each term is found by adding the same number to the previous term.

34. In a geometric sequence, each term is found by multiplying the same number by the previous term.

What is the next term in each of the following sequences?

35. 1, 5, 25, … 125
36. 7, 10, 13, … 16

Algebra: Equations and Functions

37. Complete the function table. Identify the domain and range. Then graph the function.

<table>
<thead>
<tr>
<th>x</th>
<th>2x – 1</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>−1</td>
<td>2(−1) – 1</td>
<td>−3</td>
</tr>
<tr>
<td>0</td>
<td>2(0) – 1</td>
<td>−1</td>
</tr>
<tr>
<td>1</td>
<td>2(1) – 1</td>
<td>1</td>
</tr>
</tbody>
</table>

Domain = \{-1, 0, 1\}

Range = \{-3, −1, 1\}
ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
  • You are probably ready for the Chapter Test.
  • You may want to take the Chapter 1 Practice Test on page 75 of your textbook as a final check.

☐ I used my Foldables or Study Notebook to complete the review of all or most lessons.
  • You should complete the Chapter 1 Study Guide and Review on pages 70–74 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may want to take the Chapter 1 Practice Test on page 75 of your textbook.

☐ I asked for help from someone else to complete the review of all or most lessons.
  • You should review the examples and concepts in your Study Notebook and Chapter 1 Foldables.
  • Then complete the Chapter 1 Study Guide and Review on pages 70–74 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 1 Practice Test on page 75 of your textbook.

[Signatures]
Integers

Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with two sheets of $8\frac{1}{2} \times 11$ paper.

**STEP 1**
Fold one sheet in half from top to bottom. Cut along fold from edges to margin.

**STEP 2**
Fold the other sheet in half from top to bottom. Cut along fold between margins.

**STEP 3**
Insert first sheet through second sheet and align folds.

**STEP 4**
Label each page with a lesson number and title.

**NOTE-TAKING TIPS:** When you take notes, it is helpful to list ways in which the subject matter relates to daily life.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 2. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>absolute value</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>additive inverse</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>coordinate plane</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>graph</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>integer [IHN-tih-juhr]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>negative integer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>opposites</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition</td>
<td>Description or Example</td>
</tr>
<tr>
<td>---------------------</td>
<td>---------------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>ordered pair</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>origin</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>positive integer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>quadrant</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x-axis</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x-coordinate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y-axis</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y-coordinate</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Integers and Absolute Value

**Main Idea**
- Read and write integers, and find the absolute value of a number.

**Build Your Vocabulary** (pages 32–33)

An integer is any number from the set \{…, -4, -3, -2, -1, 0, 1, 2, 3, 4, …\}.

To graph a point on the number line, draw a point on the line at its location.

Negative integers are integers less than zero.

Positive integers are integers greater than zero.

**Examples** Write Integers for Real-Life Situations

Write an integer for each situation.

1. a total rainfall of 2 inches below normal

   Because it represents below normal, the integer is \(-2\).

2. a seasonal snowfall of 3 inches above normal

   Because it represents above normal, the integer is \(+3\).

**Check Your Progress** Write an integer for each situation.

a. a total snowfall of 5 inches above normal

   \(+5\)

b. an average monthly temperature of 4 degrees below normal

   \(-4\)
The numbers \(-5\) and 5 are the same distance from 0, so \(-5\) and 5 have the same absolute value.

**EXAMPLE**  Graph integers

Graph the set of integers \{-1, 3, -2\} on a number line.

Draw a number line. Then draw a dot at the location of each integer.

**Check Your Progress**  Graph the set of integers \{-2, 1, -4\} on a number line.

**EXAMPLES**  Evaluate Expressions

1. Evaluate the expression \(|-5|\).

On the number line, the graph of \(-5\) is 5 units from 0.

So, \(|-5| = 5\).

5. Evaluate the expression \(|-4| - |-3|\).

\[|-4| - |-3| = \begin{array}{c} 4 \\ -3 \end{array} \]

\[= 1\]

Subtract.

**Check Your Progress**  Evaluate each expression.

a. \(|-9|\)  9  
b. \(|8| - |-5|\)  3

**KEY CONCEPT**  Absolute Value

The absolute value of an integer is the distance the number is from zero on a number line.

**HOMEWORK ASSIGNMENT**
Comparing and Ordering Integers

**EXAMPLE** Compare Integers

Replace the $\boxed{\leq}$ with $<$ or $>$ to make $-9 \leq -5$ a true sentence.

Graph each integer on a number line.

![Number line](image1)

Since $-9$ is to the left of $-5$, $-9 < -5$.

**Check Your Progress** Replace the $\boxed{\leq}$ with $<$ or $>$ to make $-3 \leq -6$ a true sentence.

$-3 > -6$

**EXAMPLE** Order Integers

**TEST EXAMPLE** The lowest temperatures in Europe, Greenland, Oceania, and Antarctica are listed in the table. Which list shows the temperatures in order from coolest to warmest?

<table>
<thead>
<tr>
<th>Continent</th>
<th>Record Low Temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Europe</td>
<td>$-67$</td>
</tr>
<tr>
<td>Greenland</td>
<td>$-87$</td>
</tr>
<tr>
<td>Oceania</td>
<td>14</td>
</tr>
<tr>
<td>Antarctica</td>
<td>$-129$</td>
</tr>
</tbody>
</table>

Source: *The World Almanac*

A $-67, -87, 14, -129$

B $14, -67, -87, -129$

C $-129, -87, -67, 14$

D $-67, -87, -129, 14$

(continued on the next page)
Read the Item
To order the integers, graph them on a number line.

Solve the Item
Order the integers from least to greatest by reading from left to right. The order from least to greatest is $-129, -87, -67, 14$. The answer is C.

Check Your Progress  MULTIPLE CHOICE The lowest temperatures on a given day in four cities in the United States are listed in the table. Which of the following lists the temperatures in order from coolest to warmest?

<table>
<thead>
<tr>
<th>City</th>
<th>Low Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portland, OR</td>
<td>-12</td>
</tr>
<tr>
<td>New York City, NY</td>
<td>-6</td>
</tr>
<tr>
<td>Denver, CO</td>
<td>7</td>
</tr>
<tr>
<td>Newport, RI</td>
<td>-3</td>
</tr>
</tbody>
</table>

F  $-3, -6, 7, 12$  H  $-12, 7, -6, -3$
G  $-12, -6, -3, 7$  J  $-3, -6, 7, -12$

Check Your Progress  MULTIPLE CHOICE The lowest temperatures on a given day in four cities in the United States are listed in the table. Which of the following lists the temperatures in order from coolest to warmest?

<table>
<thead>
<tr>
<th>City</th>
<th>Low Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portland, OR</td>
<td>-12</td>
</tr>
<tr>
<td>New York City, NY</td>
<td>-6</td>
</tr>
<tr>
<td>Denver, CO</td>
<td>7</td>
</tr>
<tr>
<td>Newport, RI</td>
<td>-3</td>
</tr>
</tbody>
</table>

F  $-3, -6, 7, 12$  H  $-12, 7, -6, -3$
G  $-12, -6, -3, 7$  J  $-3, -6, 7, -12$
A coordinate plane is a plane in which a horizontal number line and a vertical number line intersect at their zero points.

The horizontal number line of a coordinate plane is called the $x$-axis.

The vertical number line of a coordinate plane is called the $y$-axis.

The origin is the point at which the number lines intersect in a coordinate grid.

An ordered pair is a pair of numbers such as $(5, -2)$ used to locate a point in the coordinate plane. The $x$-coordinate is the first number. The $y$-coordinate is the second number.

**Example** Naming Points Using Ordered Pairs

1. Write the ordered pair that corresponds to point $R$. Then state the quadrant in which the point is located.

   - Start at the origin.
   - Move left to find the $x$-coordinate of point $R$, which is $-2$.
   - Move up to find the $y$-coordinate, which is $4$.

So, the ordered pair for point $R$ is $(-2, 4)$. Point $R$ is located in Quadrant II.
**Write It**

When no numbers are shown on the $x$- or $y$-axis, how long is each interval?

When no numbers are shown on the $x$- or $y$-axis, how long is each interval?

**Check Your Progress**

Write the ordered pair that names point $M$. Then name the quadrant in which the point is located.

$(3, -1)$; Quadrant IV

**Examples**

**Graph an Ordered Pair**

1. Graph and label the point $M(3, 5)$.
   - Draw a coordinate plane.
   - Start at the origin.
   - Move 3 units to the right.
   - Then move 5 units up.
   - Draw a dot and label it $M(3, 5)$.

2. Graph and label the point $G(-2, -4)$.

**Build Your Vocabulary**

The coordinate plane is separated into four sections called quadrants.

<table>
<thead>
<tr>
<th>Quadrant</th>
<th>$(+, +)$</th>
<th>$(-, +)$</th>
<th>$(+, -)$</th>
<th>$(-, -)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadrant I</td>
<td>Quadrant II</td>
<td>Quadrant III</td>
<td>Quadrant IV</td>
<td></td>
</tr>
</tbody>
</table>
Identify Quadrants

GEOGRAPHY Use the map of Utah shown below.

In which quadrant is Vernal located?

Vernal is located in the upper right quadrant.

Quadrant I.

Which of the cities labeled on the map of Utah is located in quadrant IV?

Quadrant IV is the bottom right quadrant. So, Bluff is in Quadrant IV.

Check Your Progress Refer to the map of Utah shown above.

a. In which quadrant is Tremonton located?

Quadrant II

b. Which of the cities labeled on the map of Utah shown above is located in Quadrant III?

Cedar City
Adding Integers

**Main Idea**
- Add integers.

**Examples**

**Add Integers with the Same Sign**

1. Find \(-6 + (-3)\).

   Use a number line.
   - Start at 0.
   - Move 6 units left to show \(-6\).
   - From there, move 3 units left to show \(-3\).

   ![Number Line Diagram](image)

   So, \(-6 + (-3) = -9\).

2. Find \(-34 + (-21)\).

   \(-34 + (-21) = -55\)  
   Both integers are negative, so the sum is negative.

**Check Your Progress**

Find each sum.

a. \(-5 + (-2)\)

b. \(-27 + (-19)\)

-7

-46

**Key Concepts**

Adding Integers with the Same Sign: The sum of two positive integers is positive. The sum of two negative integers is negative.

Additive Inverse Property: The sum of any number and its additive inverse is 0.

**Build Your Vocabulary**

The integers 5 and \(-5\) are called opposites of each other because they are the same distance from 0, but on opposite sides of 0.

Two integers that are opposites are also called additive inverses.
**KEY CONCEPT**

**Adding Integers with Different Signs**

To add integers with different signs, subtract their absolute values. The sum is:

- positive if the positive integer has the greater absolute value.
- negative if the negative integer has the greater absolute value.

**EXAMPLES**

**Find 8 + (−7).**

Use a number line.

Start at 0.

Move 8 units right.

Then move 7 units left.

So, \(8 + (−7) = 1\).

**Find −5 + 4.**

Use a number line.

Start at 0.

Move 5 units left.

Then move 4 units right.

So, \(−5 + 4 = −1\).

**Check Your Progress**

Add.

a. \(6 + (−2)\)  
   b. \(−3 + 5\)
**EXAMPLES**  Add Integers with Different Signs

5. Find $2 + (-7)$.

$2 + (-7) = -5$

Subtract absolute values; $7 - 2 = 5$. Since $-7$ has the greater absolute value, the sum is negative.


$-9 + 6 = -3$

Subtract the absolute values; $9 - 6 = 3$. Since $-9$ has the greater absolute value, the sum is negative.

**Check Your Progress**  Add.

a. $5 + (-9)$  
   $-4$

b. $7 + (-3)$  
   $4$

**EXAMPLE**  Use the Additive Inverse Property

7. Find $11 + (-4) + (-11)$.

$11 + (-4) + (-11) = 11 + (-11) + (-4)$  

Commutative Property (+)  

$= 0 + (-4)$  

Additive Inverse Property  

$= -4$  

Identity Property (+)

**Check Your Progress**  Find $5 + (-11) + (-5)$.

$-11$
EXAMPLES

Subtract Positive Integers

1. Find $2 - 15$.
   
   $2 - 15 = 2 + (-15)$
   
   To subtract 15, add $-15$.
   
   $= -13$
   
   Simplify.

   
   $-13 - 8 = -13 + (-8)$
   
   To subtract 8, add $-8$.
   
   $= -21$
   
   Simplify.

Check Your Progress

Subtract.

a. $13 - 21$
   
   $-8$

b. $-9 - 11$
   
   $-20$

EXAMPLES

Subtract Negative Integers

3. Find $12 - (-6)$.
   
   $12 - (-6) = 12 + 6$
   
   To subtract $-6$, add 6.
   
   $= 18$
   
   Simplify.

4. Find $-21 - (-8)$.
   
   $-21 - (-8) = -21 + 8$
   
   To subtract $-8$, add 8.
   
   $= -13$
   
   Simplify.

Check Your Progress

Subtract.

a. $9 - (-4)$
   
   13

b. $17 - (-6)$
   
   23
EXAMPLE Evaluate an Expression

## ALGEBRA
Evaluate $g - h$ if $g = -2$ and $h = -7$.

$$ g - h = \boxed{-2} - \boxed{(-7)} $$

Replace $g$ with $-2$ and $h$ with $-7$.

$$ = -2 + \boxed{7} $$

Subtract $-7$, add $7$.

$$ = \boxed{5} $$

Simplify.

**Check Your Progress** Evaluate $m - n$ if $m = -6$ and $n = 4$.

$-10$

---

**WRITE IT**

Explain how you can use a number line to check the results of subtracting integers.

---

**EXAMPLE**

## GEOGRAPHY
In Mongolia, the temperature can fall to $-45^\circ C$ in January. The temperature in July may reach $40^\circ C$. What is the difference between these two temperatures in Mongolia?

To find the difference in temperatures, subtract the lower temperature from the higher temperature.

$$ 40 - (-45) = \boxed{40} + \boxed{45} $$

To subtract $-45$, add $45$.

$$ = \boxed{85} $$

Simplify.

So, the difference between the temperatures is $85^\circ C$.

**Check Your Progress** On a particular day in Anchorage, Alaska, the high temperature was $15^\circ F$ and the low temperature was $-11^\circ F$. What is the difference between these two temperatures for that day?

$26^\circ F$
2–6 Multiplying Integers

**Main Idea**
- Multiply integers.

**Key Concepts**

**Multiply Integers with Different Signs**
The product of two integers with different signs is negative.

**Multiply Integers with the Same Sign**
The product of two integers with the same sign is positive.

**Examples**

1. **Multiply Integers with Different Signs**
   - Find $5(-4)$.
     - $5(-4) = -20$
     - The integers have **different** signs.
     - The product is **negative**.
   - Find $-3(9)$.
     - $-3(9) = -27$
     - The integers have **different** signs.
     - The product is **negative**.

2. **Multiply Integers with the Same Sign**
   - Find $-6(-8)$.
     - $-6(-8) = 48$
     - The integers have the **same** sign.
     - The product is **positive**.
   - Find $(-8)^2$.
     - $(-8)^2 = (-8)(-8)$
     - There are **two** factors of $-8$.
     - $= 64$
     - The product is **positive**.
   - Find $-2(-5)(-6)$.
     - $-2(-5)(-6) = 10(-6) = -2(-5) = 10$
     - $= -60$
     - $10(-6) = -60$
Check Your Progress  Multiply.

a. $-4(-7)$  
   \[
   -4(-7) = 28
   \]

b. $(-5)^2$  
   \[
   (-5)^2 = 25
   \]

c. $-7(-3)(-4)$  
   \[
   -7(-3)(-4) = -84
   \]

Example

Mines A mine elevator descends at a rate of 300 feet per minute. How far below the earth’s surface will the elevator be after 5 minutes?

If the elevator descends 300 feet per minute, then after 5 minutes, the elevator will be $-300 \times 5$ or $-1,500$ feet below the surface. Thus, the elevator will descend to $-1,500$ feet.

Check Your Progress  Retirement Mr. Rodriguez has $78 deducted from his pay every month and placed in a savings account for his retirement. What integer represents a change in his savings account for these deductions after six months?

$468$

Example

Evaluate Expressions

ALGEBRA  Evaluate $abc$ if $a = -3$, $b = 5$, and $c = -8$.

\[
abc = (-3)(5)(-8)
\]

Replace $a$ with $-3$, $b$ with $5$, and $c$ with $-8$.

\[
= (-15)(-8)
\]

Multiply $-3$ and $5$.

\[
= 120
\]

Check Your Progress  Evaluate $xyz$ if $x = -6$, $y = -2$, and $z = 4$.

$48$
Problem-Solving Investigation:
Look for a Pattern

EXAMPLE  Use the Look for a Pattern Strategy

HAIR  Lelani wants to grow an 11-inch ponytail. She has a 3-inch ponytail now, and her hair grows about one inch every two months. How long will it take for her ponytail to reach 11 inches?

UNDERSTAND  You know the length of Lelani’s ponytail now. You know how long Lelani wants her ponytail to grow and you know how fast her hair grows. You need to know how long it will take for her ponytail to reach 11 inches.

PLAN  Look for a pattern. Then extend the pattern to find the solution.

SOLVE  After the first two months, Lelani’s ponytail will be 3 inches + 1 inch, or 4 inches. Every two months, her hair grows according to the pattern below.

\[
\begin{align*}
3 \text{ in.} & \quad 4 \text{ in.} & \quad 5 \text{ in.} & \quad 6 \text{ in.} & \quad 7 \text{ in.} & \quad 8 \text{ in.} & \quad 9 \text{ in.} & \quad 10 \text{ in.} & \quad 11 \text{ in.} \\
+1 & \quad +1 & \quad +1 & \quad +1 & \quad +1 & \quad +1 & \quad +1 & \quad +1 \\
\end{align*}
\]

It will take eight sets of two months, or 16 months total, for Lelani’s ponytail to reach 11 inches.

CHECK  Lelani’s ponytail grew from 3 inches to 11 inches, a difference of eight inches, in 16 months. Since one inch of growth requires two months and \(8 \times 2 = 16\), the answer is correct.

Check Your Progress  RUNNING  Samuel ran 2 miles on his first day of training to run a marathon. On the third day, Samuel increased the length of his run by 1.5 miles. If this pattern continues every three days, how many miles will Samuel run on the 27th day?

16.5 miles
Dividing Integers

**EXAMPLE** Dividing Integers with Different Signs

1. Find $51 \div (-3)$.

$$51 \div (-3) = -17$$

The integers have different signs. The quotient is negative.

2. Find $\frac{-121}{11}$.

$$\frac{-121}{11} = -11$$

The integers have different signs. The quotient is negative.

**EXAMPLE** Dividing Integers with the Same Sign

3. Find $-12 \div (-2)$.

$$-12 \div (-2) = 6$$

The integers have the same sign. The quotient is positive.

**Check Your Progress** Find each quotient.

a. $36 \div (-9)$  
   -4

b. $\frac{45}{-9}$  
   -5

c. $-24 \div (-8)$  
   3

**EXAMPLE** ALGEBRA Evaluate $-18 \div x$ if $x = -2$.

$$-18 \div x = -18 \div \boxed{2}$$

Replace $x$ with $-2$.

$$= \boxed{9}$$

Divide. The quotient is negative.

**Check Your Progress** ALGEBRA Evaluate $g \div h$ if $g = 21$ and $h = -3$.

-7
2-1 Integers and Absolute Value

Express each of the following in words.

1. +7  positive seven
2. −7  negative seven
3. |7|  absolute value of seven

4. On the following number line, draw an oval around the negative integers and label them negative. Draw a rectangle around the positive integers and label them positive.

2-2 Comparing and Ordering Integers

Write each expression in words.

5. −1 < 0  negative one is less than zero
6. 3 > −2  three is greater than negative two
The Coordinate Plane

Look at the coordinate plane at the right. Name the ordered pair for each point graphed.

7. $A (-3, 3)$
8. $B (3, 2)$
9. $C (3, -2)$

In the coordinate plane above, identify the quadrant in which each lies.

10. $A$ Quadrant II
11. $B$ Quadrant I
12. $C$ Quadrant IV

Adding Integers

Tell how you would solve each of the following on a number line, then add.

13. $-7 + (-9)$ Start at 0. Move left 7 units. Then move left 9 units; $-16$
14. $-7 + 9$ Start at 0. Move left 7 units. Then move left another 9 units; 2

15. How many units away from 0 is the number 17? 17
16. How many units away from 0 is the number $-17$? 17
17. What are 17 and $-17$ called? opposites or additive inverses
Subtracting Integers

Find each difference. Write an equivalent addition sentence for each.

18. \(1 - 5\) \(-4; 1 + (-5) = -4\)
19. \(-2 - 1\) \(-3; -2 + (-1) = -3\)
20. \(-3 - 4\) \(-7; -3 + (-4) = -7\)

Multiplying Integers

Choose the correct term to complete each sentence.

21. The product of two integers with different signs is \(\text{positive, negative}\).
22. The product of two integers with the same sign is \(\text{positive, negative}\).

Find each product.

23. \((-6)(-4)\) \(24\)
24. \(-8(5)\) \(-40\)
25. \(-2(3)(-4)\) \(24\)

Problem-Solving Investigation: Look for a Pattern

26. CANS A display of soup cans at the end of a store aisle contains 1 can in the top row and 2 cans in each additional row beneath it. If there are 6 rows in the display, how many cans are in the sixth row?

\(11\)

Dividing Integers

Write two division sentences for each of the following multiplication sentences.

27. \(6(-3) = 18\) \(18 \div (-3) = -6\) \(18 \div (-6) = -3\)
28. \(-21(-2) = 42\) \(42 \div (-2) = -21\) \(42 \div (-2) = -21\)
ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
  • You are probably ready for the Chapter Test.
  • You may want to take the Chapter 2 Practice Test on page 123 of your textbook as a final check.

☐ I used my Foldables or Study Notebook to complete the review of all or most lessons.
  • You should complete the Chapter 2 Study Guide and Review on pages 119–122 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may want to take the Chapter 2 Practice Test on page 123 of your textbook.

☐ I asked for help from someone else to complete the review of all or most lessons.
  • You should review the examples and concepts in your Study Notebook and Chapter 2 Foldables.
  • Then complete the Chapter 2 Study Guide and Review on pages 119–122 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 2 Practice Test on page 123 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 2.
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with a sheet of 11" × 17" paper.

**STEP 1** Fold the short sides toward the middle.

**STEP 2** Fold the top to the bottom.

**STEP 3** Open. Cut along the second fold to make four tabs.

**STEP 4** Label each of the tabs as shown.

**NOTE-TAKING TIP:** When you take notes, listen or read for main ideas. Then record those ideas in a simplified form for future reference.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 3. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition Property of Equality</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Division Property of Equality</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>formula</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>linear equation</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(continued on the next page)
<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subtraction Property of Equality</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>two-step equation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>work backward strategy</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3–1 Writing Expressions and Equations

**Main Idea**
- Write verbal phrases and sentences as simple algebraic expressions and equations.

**Foldables**
Write two phrases and their algebraic expressions under the Expressions tab.

**Organize It**

**EXAMPLE** Write a Phrase as an Expression

1. Write the phrase *twenty dollars less the price of a movie ticket* as an algebraic expression.

<table>
<thead>
<tr>
<th>Words</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>twenty dollars less the price of a movie ticket</td>
<td>20 – m</td>
</tr>
</tbody>
</table>

   Let \( m \) = the price of a movie ticket.

**Check Your Progress** Write the phrase *five more inches of snow than last year’s snowfall* as an algebraic expression.

\[ s + 5 \]

**Examples** Write Sentences as Equations

Write each sentence as an algebraic equation.

2. A number less 4 is 12.

<table>
<thead>
<tr>
<th>Words</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A number less 4 is 12.</td>
<td>4 - n = 12</td>
</tr>
</tbody>
</table>

   Let \( n \) represent a number.

3. Twice a number is 18.

<table>
<thead>
<tr>
<th>Words</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Twice a number is 18.</td>
<td>2a = 18</td>
</tr>
</tbody>
</table>

   Let \( a \) represent a number.
Check Your Progress  Write each sentence as an algebraic equation.

a. Eight less than a number is 12.

\[ n - 8 = 12 \]

b. Four times a number equals 96.

\[ 4x = 96 \]

EXAMPLE

FOOD An average American adult drinks more soft drinks than any other beverage each year. Three times the number of gallons of soft drinks plus 27 is equal to the total 183 gallons of beverages consumed. Write the equation that models this situation.

Three times the number of gallons of soft drinks plus 27 equals 183.

\[ 3s + 27 = 183 \]

Let \( s \) = the number of gallons of soft drinks.

Check Your Progress  EXERCISE It is estimated that American adults spend an average of 8 hours per month exercising. This is 26 hours less than twice the number of hours spent watching television each month. Write an equation that models this situation.

\[ 2t - 26 = 8 \]
Solving Addition and Subtraction Equations

**EXAMPLES**

**Solve an Addition Equation**


   $$14 + y = 20$$
   Write the equation.

   $$-14 -14$$
   Subtract 14 from each side. Simplify.

   $$y = 6$$

   **Check**

   $$14 + y = 20$$
   Write the original equation.

   $$14 + 6 \neq 20$$
   Replace $y$ with 6.

   $$20 = 20 \checkmark$$
   Simplify.

   The solution is 6.

2. Solve $a + 7 = 6$. Check your solution.

   $$a + 7 = 6$$
   Write the equation.

   $$-7 -7$$
   Subtract 7 from each side.

   $$a = -1$$

   **Check**

   $$a + 7 = 6$$
   Write the original equation.

   $$-1 + 7 \neq 6$$
   Replace $a$ with $-1$.

   $$6 = 6 \checkmark$$
   Simplify. The solution is $-1$.

**Check Your Progress**

Solve each equation.

a. $-6 = x + 4$

b. $m + 9 = 22$

$-10$

13
FRUIT  A grapefruit weighs 11 ounces, which is 6 ounces more than an apple. How much does the apple weigh?

<table>
<thead>
<tr>
<th>Words</th>
<th>A grapefruit’s  is 6 more than an apple’s weight.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>Let ( a ) represent the apple’s weight.</td>
</tr>
<tr>
<td>Equation</td>
<td>[ 11 = 6 + a ]</td>
</tr>
</tbody>
</table>

Write the equation.

\[ 11 = 6 + a \]

Subtract 6 from each side.

\[ -6 \]

Simplify.

\[ 5 = a \]

The apple weighs 5 ounces.

Check Your Progress  EXERCISE  Cedric ran 17 miles this week, which is 9 more miles than he ran last week. How many miles did he run last week?

8 miles

EXAMPLE  Solve a Subtraction Equation

Solve \( 12 = z - 8 \).

\[ 12 = z - 8 \]

Write the equation.

Add 8 to each side.

\[ +8 \]

Simplify.

\[ 20 = z \]

The solution is 20.

Check Your Progress  Solve \( w - 5 = 27 \).

32
EXAMPLES

Solving Multiplication Equations

1. Solve $39 = 3y$. Check your solution.

   \[ 39 = 3y \]

   Write the equation.

   \[ \frac{39}{3} = \frac{3y}{3} \]

   Divide each side of the equation by 3.

   \[ 13 = y \]

   \[ 39 \div 3 = 13 \]

   Check

   \[ 39 = 3y \]

   Write the equation.

   \[ 39 \div 3 (13) \]

   Replace $y$ with 13. Is this sentence true?

   \[ 39 = 39 \checkmark \]

   So, the solution is 13.

2. Solve $-4z = 60$. Check your solution.

   \[ -4z = 60 \]

   Write the equation.

   \[ \frac{-4z}{-4} = \frac{60}{-4} \]

   Divide each side of the equation by $-4$.

   \[ z = -15 \]

   \[ 60 \div (-4) = -15 \]

   Check

   \[ -4z = 60 \]

   Write the equation.

   \[ -4(-15) = 60 \]

   Replace $z$ with $-15$. Is this sentence true?

   \[ 60 = 60 \checkmark \]

   So, the solution is $-15$. 

Main Idea

- Solve multiplication equations.

Key Concept

Division Property of Equality: If you divide each side of an equation by the same nonzero number, the two sides remain equal.

Foldables

Record the Division Property of Equality in your own words under the Equation tab.
Check Your Progress  Solve each equation. Check your solution.

a. \(6m = 42\)  
   \[7\]  
   

b. \(-64 = -16b\)
   
   \[4\]

Build Your Vocabulary (pages 55–56)

A formula is an equation that shows the relationship among certain quantities.

Example

Swimming  Ms. Wang swims at a speed of 0.6 mph. At this rate, how long will it take her to swim 3 miles? (parallels Example 4 in text)

You are asked to find the time \(t\) it will take to swim a distance \(d\) of 3 miles at a rate \(r\) of 0.6 mph.

\[d = rt\]  
Write the equation.

\[3 = 0.6t\]  
Replace \(d\) with 3 and \(r\) with 0.6.

\[
\frac{3}{0.6} = \frac{0.6t}{0.6} \\
5 = t  \\
3 \div 0.6 = 5
\]

It would take Ms. Wang 5 hours to swim 3 miles.

Check Your Progress  Cookies  Debbie spends $6.85 on cookies at the bakery. The cookies are priced at $2.74 per pound. How many pounds of cookies did Debbie buy?

\[2.5\]
EXAMPLE Use the Work Backward Strategy

SHOPPING Lucy and Elena went to the mall. Each girl bought a CD for $16.50, a popcorn for $3.50, and a drink for $2.50. Altogether, they had $5.00 left over. How much money did they take to the mall?

UNDERSTAND You know that they had $5.00 left over and how much they spent on each item. You need to know how much they took to the mall.

PLAN Start with the end result and work backward.

SOLVE They had $5.00 left.

Undo the two drinks for $2.50 each.

\[5 + 2(2.50) = 10\]

Undo the two popcorns for $3.50 each.

\[10 + 2(3.50) = 17\]

Undo the two CDs for $16.50 each.

\[17 + 2(16.50) = 50\]

So, they took $50 to the mall.

CHECK Assume they started with $50. After buying two CDs, they had $50 - 2($16.50) or $17. After buying two pop corns, they had $17 - 2($3.50) or $10. After buying two drinks, they had $10 - 2($2.50) or $5. So, the answer is correct.

Check Your Progress AIRPORT Jack needs to go home from work to pack before heading to the airport. He wants to be at the airport by 1:15 P.M. It takes him 20 minutes to drive home from work, 30 minutes to pack, and 45 minutes to get to the airport from home. What time should he leave work?

11:40 A.M.
Main Idea
- Solve two-step equations.

Build Your Vocabulary (pages 55–56)
A two-step equation has two different operations.

Examples
Solve Two-Step Equations

1 Solve $4x + 3 = 19$. Check your solution.

Write the equation.

\[ 4x + 3 = 19 \]

Subtract 3 from each side.

\[
\begin{array}{c}
4x \\
-3 \\
\hline
4x
\end{array}
\]

Simplify.

\[ 4x = 16 \]

Divide each side by 4.

\[
\begin{array}{c}
\frac{4x}{4} \\
\frac{16}{4} \\
\hline
\frac{4x}{4} = \frac{16}{4}
\end{array}
\]

Simplify.

\[ \frac{4x}{4} = \frac{16}{4} \]

\[ x = 4 \]

Check

\[ 4x + 3 = 19 \]

Write the original equation.

\[ 4 \left( \frac{4}{4} \right) + 3 = 19 \]

Replace x with 4.

\[ 16 + 3 = 19 \]

Simplify.

\[ 19 = 19 \checkmark \]

The solution is 4.

2 Solve $6 + 5y = 26$.

Write the equation.

\[ 6 + 5y = 26 \]

Subtract 6 from each side.

\[
\begin{array}{c}
6 \\
-6 \\
\hline
5y
\end{array}
\]

Simplify.

\[ 5y = 20 \]

Divide each side by 5.

\[
\begin{array}{c}
\frac{5y}{5} \\
\frac{20}{5} \\
\hline
\frac{5y}{5} = \frac{20}{5}
\end{array}
\]

Simplify.

\[ \frac{5y}{5} = \frac{20}{5} \]

\[ y = 4 \]
3–5 Solve \(-3c + 9 = 3\).

\[
\begin{align*}
-3c + 9 &= 3 \\
-9 &= -9 \\
\hline
-3c &= -6 \\
\frac{-3c}{-3} &= \frac{-6}{-3} \\
\hline
c &= 2
\end{align*}
\]

The solution is \(2\).

4 Solve \(0 = 6 + 3t\).

\[
\begin{align*}
0 &= 6 + 3t \\
-6 &= -6 \\
\hline
-6 &= 3t \\
\frac{-6}{3} &= \frac{3t}{3} \\
\hline
-2 &= t
\end{align*}
\]

The solution is \(-2\).

**Check Your Progress** Solve each equation.

**a.** \(3t - 7 = 14\)

\[
\begin{align*}
7
\end{align*}
\]

**b.** \(4 + 2w = 18\)

\[
\begin{align*}
7
\end{align*}
\]

**c.** \(-8k + 7 = 31\)

\[
\begin{align*}
-3
\end{align*}
\]

**d.** \(0 = -4x + 32\)

\[
\begin{align*}
8
\end{align*}
\]
EXAMPLE PARKS There are 76 thousand acres of state parkland in Georgia. This is 4 thousand acres more than three times the number of acres of state parkland in Mississippi. How many acres of state parkland are there in Mississippi?

Three times the number of acres of state parkland in Mississippi plus 4,000 is 76,000.

Let \( m \) = the acres of state parkland in Mississippi.

Three times the number of acres of parkland in Mississippi plus 4,000 is 76,000.

\[
3m + 4,000 = 76,000
\]

Write the equation.

\[
-4,000 - 4,000
\]

Subtract 4,000 from each side.

\[
3m = 72,000
\]

Simplify.

\[
\frac{3m}{3} = \frac{72,000}{3}
\]

Divide each side by 3.

\[
m = 24,000
\]

Simplify.

There are 24,000 acres of state parkland in Mississippi.

Check Your Progress BASEBALL Matthew had 64 hits during last year’s baseball season. This was 8 less than twice the number of hits Gregory had. How many hits did Gregory have during last year’s baseball season?
The distance around a geometric figure is called the **perimeter**.

**BUILD YOUR VOCABULARY**

**Example 1** Find the Perimeter of a Rectangle

Find the perimeter of the rectangle.

\[ P = 2\ell + 2w \]

**Perimeter of a rectangle**

\[ P = 2(18) + 2(2) \]

\[ \ell = 18, \ w = 2 \]

\[ P = 36 + 4 \]

Multiply.

\[ P = 40 \]

Add.

The perimeter is 40 **feet**.

**Check Your Progress** Find the perimeter of a rectangle with a length of 2.35 centimeters and a width of 11.9 centimeters.

28.5 cm

**Example 2**

**ART** A painting has a perimeter of 68 inches. If the width of the painting is 13 inches, what is its length?

\[ P = 2\ell + 2w \]

**Perimeter of a rectangle**

\[ 68 = 2\ell + 2(13) \]

Replace \( P \) with 68 and \( w \) with 13.

\[ 68 = 2\ell + 26 \]

Multiply.

(continued on the next page)
68 − 26 = 2\ell + 26 − 26
Subtract 26 from each side.

\[
\begin{align*}
42 &= 2\ell \\
21 &= \ell
\end{align*}
\]
Simplify.
Divide each side by 2.

**Check Your Progress**  
**GARDENS** A tomato garden has a perimeter of 22.2 feet. If the length of the garden is 6.3 feet, find the width.

4.8 ft

**BUILD YOUR VOCABULARY** (pages 55–56)

The area is the measure of the surface enclosed by a figure.

**EXAMPLE**  
**Find The Area of a Rectangle**

**FRESHWATER** Find the area of the surface of the reservoir shown below.

Area of a Rectangle
The area \( A \) of a rectangle is the product of the length \( \ell \) and width \( w \).

\[ A = \ell \cdot w \]

\[ A = 4 \cdot 0.625 \]
Replace \( \ell \) with 4 and \( w \) with 0.625.

\[ A = 2.5 \]
Multiply.

The area is 2.5 square miles.

**Check Your Progress**  
**PAINTING** Sue is painting a wall that measures 18.25 feet long and 8 feet high. Find the area of the surface Sue will be painting.

146 ft²
**EXAMPLE**

**Work** The table shows the number of hours Abby worked and her corresponding earnings. Make a graph of the data to show the relationship between the number of hours Abby worked and her earnings.

The ordered pairs $(1, 6), (2, 12), (3, 18),$ and $(4, 24)$ represent the function. Graph the ordered pairs.

<table>
<thead>
<tr>
<th>Number of Hours</th>
<th>Earnings ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
</tr>
</tbody>
</table>

**Check Your Progress** **VIDEOS** Make a graph of the data in the table that shows the relationship between the amount David would pay and the number of movies he rents.

<table>
<thead>
<tr>
<th>Number of Videos</th>
<th>Amount ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$3.50</td>
</tr>
<tr>
<td>2</td>
<td>$7.00</td>
</tr>
<tr>
<td>3</td>
<td>$10.50</td>
</tr>
<tr>
<td>4</td>
<td>$14.00</td>
</tr>
</tbody>
</table>
An equation like $y = 2x + 1$ is a **linear equation** because the graph is a **straight** line.

**Example**

**Graph Solutions of Linear Equations**

Graph $y = x + 3$.

Select any four values for the input $x$. We chose 2, 1, 0, and $-1$. Substitute these values for $x$ to find the output $y$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$x + 3$</th>
<th>$y$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2 + 3</td>
<td>5</td>
<td>(2, 5)</td>
</tr>
<tr>
<td>1</td>
<td>1 + 3</td>
<td>4</td>
<td>(1, 4)</td>
</tr>
<tr>
<td>0</td>
<td>0 + 3</td>
<td>3</td>
<td>(0, 3)</td>
</tr>
<tr>
<td>$-1$</td>
<td>$-1 + 3$</td>
<td>2</td>
<td>($-1$, 2)</td>
</tr>
</tbody>
</table>

Four solutions are $(2, 5)$, $(1, 4)$, $(0, 3)$, and $(-1, 2)$.

**Check Your Progress**

Graph $y = 3x - 2$.

Four solutions are $(-1, -5)$, $(0, -2)$, $(1, 1)$, and $(2, 4)$. 
EXAMPLE Represent Real-World Functions

ANIMALS Blue whales can reach a speed of 30 miles per hour. The equation \( d = 30t \) describes the distance \( d \) that a whale swimming at that speed can travel in time \( t \). Assuming that a whale can maintain that speed, represent the function with a graph.

**Step 1** Select four values for \( t \). Select only positive numbers since \( t \) represents time. Make a function table.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{ } & \text{30}t & \text{ } & (t, d) \\
\hline
2 & 30(2) & \boxed{60} & (2, 60) \\
\hline
3 & 30(3) & 90 & (3, 90) \\
\hline
5 & 30(5) & \boxed{150} & (5, 150) \\
\hline
6 & 30(6) & 180 & (6, 180) \\
\hline
\end{array}
\]

**Step 2** Graph the ordered pairs and draw a line through the points.

**Check Your Progress** TRAVEL Susie takes a car trip traveling at an average speed of 55 miles per hour. The equation \( d = 55t \) describes the distance \( d \) that Susie travels in time \( t \). Represent this function with a graph.
3-1 Writing Expressions and Equations

Match the phrases with the algebraic expressions that represent them.

1. seven plus a number
   - seven less a number
   - seven divided by a number
   - seven less than a number
   - a. 7 - n
   - b. 7 \cdot n
   - c. n - 7
   - d. \frac{n}{7}
   - e. 7 + n

2. seven less a number

3. seven divided by a number

4. seven less than a number

Write each sentence as an algebraic equation.

5. The product of 4 and a number is 12.
   - 4n = 12

6. Twenty divided by y is equal to -10.
   - \frac{20}{y} = -10

3-2 Solving Addition and Subtraction Equations

7. Explain in words how to solve a - 10 = 3.
   - Add ten to each side and then simplify.

Solve each equation. Check your solution.

8. w + 23 = -11
   - -34

9. 35 = z - 15
   - 50
3-3
Solving Multiplication Equations

10. To solve \(-27 = -3d\), divide each side by \(-3\).

Solve each equation. Check your solution.
11. \(36 = 6k\)  
   \(6\)

12. \(-7z = 28\)  
   \(-4\)

3-4
Problem-Solving Investigation: Work Backward

13. AGE Bradley is four years older than his brother Philip. Philip is 7 years younger than Kailey, who is 2 years older than Taneesha. If Taneesha is 11 years old, how old is Bradley?

   10 years old

3-5
Solving Two-Step Equations

14. Describe in words each step shown for solving \(12 + 7s = -9\).

\[
\begin{align*}
12 + 7s &= -9 \\
-12 &\quad -12 \\
7s &= -21 \\
\frac{7s}{7} &= \frac{-21}{7} \\
s &= -3
\end{align*}
\]

Write the equation.  
Subtract 12 from each side.  
Simplify.  
Divide each side by 7.  
Simplify.

15. Number the steps in the correct order for solving the equation \(-4v + 11 = -5\).

\[
\begin{align*}
3, 5 &\quad \text{Simplify} \\
4 &\quad \text{Divide each side by \(-4\).} \\
2 &\quad \text{Subtract 11 from each side.} \\
1 &\quad \text{Write the equation.} \\
5, 3 &\quad \text{Simplify.} \\
6 &\quad \text{Check the solution.}
\end{align*}
\]
3-6 Measurement: Perimeter and Area

Find the perimeter and area of each rectangle.

16.  
\[
\text{Perimeter} = 2(18.2\, \text{cm} + 6.7\, \text{cm}) = 49.8\, \text{cm} \\
\text{Area} = 18.2\, \text{cm} \times 6.7\, \text{cm} = 121.94\, \text{cm}^2
\]

17.  
\[
\text{Perimeter} = 2(5\, \text{yd} + 1.19\, \text{yd}) = 12.38\, \text{yd} \\
\text{Area} = 5\, \text{yd} \times 1.19\, \text{yd} = 5.95\, \text{yd}^2
\]

18. **FRAMING** Marcia wants to frame her favorite painting. If the frame is 3.25 feet wide and the perimeter is 15.7 feet, find the width of the frame.

\[
\text{Width of the frame} = \frac{15.7\, \text{ft} - 2 \times 3.25\, \text{ft}}{2} = 4.6\, \text{ft}
\]

3-7 Functions and Graphs

19. Complete the function table. Then graph the function.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(2x - 1)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>2(-1) - 1</td>
<td>-3</td>
</tr>
<tr>
<td>0</td>
<td>2(0) - 1</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>2(1) - 1</td>
<td>1</td>
</tr>
</tbody>
</table>
ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
  • You are probably ready for the Chapter Test.
  • You may want to take the Chapter 3 Practice Test on page 173 of your textbook as a final check.

☐ I used my Foldables or Study Notebook to complete the review of all or most lessons.
  • You should complete the Chapter 3 Study Guide and Review on pages 169–172 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may want to take the Chapter 3 Practice Test on page 173.

☐ I asked for help from someone else to complete the review of all or most lessons.
  • You should review the examples and concepts in your Study Notebook and Chapter 3 Foldable.
  • Then complete the Chapter 3 Study Guide and Review on pages 169–172 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 3 Practice Test on page 173.

---

Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 3.

---

Student Signature

Parent/Guardian Signature

Teacher Signature

---

Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin this Interactive Study Notebook to help you in taking notes.

Begin with five sheets of $8\frac{1}{2}'' \times 11''$ paper.

**STEP 1** Stack five sheets of paper $\frac{3}{4}$ inch apart.

**STEP 2** Roll up bottom edges so that all tabs are the same size.

**STEP 3** Crease and staple along the fold.

**STEP 4** Write the chapter title on the front. Label each tab with a lesson number and title.

**NOTE-TAKING TIP:** Before each lesson, skim through the lesson and write any questions that come to mind in your notes. As you work through the lesson, record the answer to your question.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 4. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>bar notation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>common denominator</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>composite number [kahm-PAH-zuht]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>equivalent [ih-KWIH-vuh-luhnt]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fractions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>factor tree</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>greatest common factor (GCF)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>least common denominator (LCD)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>least common multiple (LCM)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>multiple</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition</td>
<td>Description or Example</td>
</tr>
<tr>
<td>-----------------------</td>
<td>---------------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>percent</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>prime factorization</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>prime number</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ratio</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rational number</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>repeating decimal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>simplest form</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>terminating decimal</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Main Idea**

- Find the prime factorization of a composite number.

**Build Your Vocabulary** (pages 77–78)

A **prime number** is a whole number greater than 1 that has exactly **two** factors, **1** and **itself**.

A **composite number** is a whole number greater than **1** that has more than **2** factors.

Every **composite** number can be written as a product of prime numbers exactly one way called the **prime factorization**.

A **factor tree** can be used to find the factorization.

**Examples**

**Identify Numbers as Prime or Composite**

Determine whether each number is **prime or composite**.

1. **63**
   - 63 has six factors: 1, 3, 7, 9, 21, and 63.
   - So, it is **composite**.

2. **29**
   - 29 has only two factors: 1 and 29.
   - So, it is **prime**.

**Check Your Progress**

Determine whether each number is **prime or composite**.

a. **41**
   - prime

b. **24**
   - composite
EXAMPLE Find the Prime Factorization

Find the prime factorization of 100.

To find the prime factorization, you can use a factor tree or divide by prime numbers. Let’s use a factor tree.

\[ 100 = 2 \times 2 \times 5 \times 5 \text{ or } 2^2 \times 5^2. \]

EXAMPLE Find an Algebraic Expression

ALGEBRA Factor \(21m^2n\).

\[ 21m^2n = 3 \times 7 \times m \times m \times n \]

Check Your Progress

a. Find the prime factorization of 72.

\[ 2^3 \times 3^2 \]

b. Factor \(15xy^3\).

\[ 3 \cdot 5 \cdot x \cdot y \cdot y \cdot y \]
A Venn diagram uses circles to show how elements among sets of numbers or objects are related.

The greatest number that is a common factor to two or more numbers is called the greatest common factor (GCF).

**Example**

Find the Greatest Common Factor

**Find the GCF of 28 and 42.**

**Method 1** First, list the factors of 28 and 42.

Factors of 28: 1, 2, 4, 7, 14, 28

Factors of 42: 1, 2, 3, 6, 7, 14, 21, 42

The common factors are 1, 2, 7, and 14.

So, the GCF is 14.

**Method 2** Use prime factorization.

\[ 28 = 2 \times 2 \times 7 \]

\[ 42 = 2 \times 3 \times 7 \]

The greatest common factor or GCF is 2 \times 7 or 14.

**Check Your Progress** Find the GCF of 18 and 45.

9
**EXAMPLE** Find the GCF of Three Numbers

Find the GCF of 21, 42, and 63.

**METHOD 1** First, list the factors of 21, 42, and 63.

factors of 21: 1, 3, 7

factors of 42: 1, 2, 3, 6, 7, 14, 21, 42

factors of 63: 1, 3, 7, 9, 21, 63

The common factors of 21, 42, and 63 are 1, 3, and 7.

So, the greatest common factor or GCF is 7.

**METHOD 2** Use prime factorization.

21 = \(3 \times 7\)

42 = \(2 \times 3 \times 7\)  Circle the common factors.

63 = \(3 \times 3 \times 7\)

The common prime factors are 3 and 7.

The GCF is \(3 \times 7\), or 21.

---

**Check Your Progress** Find the GCF of each set of numbers.

24, 48, and 60

12

---

**EXAMPLE**

> **ART** Searra wants to cut a 15-centimeter by 25-centimeter piece of tag board into squares for an art project. She does not want to waste any of the tag board and she wants the largest squares possible. What is the length of the side of the squares she should use?
The largest length of side possible is the GCF of the dimensions of the tag board.

\[15 = 3 \times 5\]
\[25 = 5 \times 5\]

The GCF of 15 and 25 is 5. So, Searra should use squares with sides measuring 5 centimeters.

**EXAMPLE**

How many squares can she make if the sides are 5 centimeters?

\[25 \div 5 = 5\] squares can fit along the length.
\[15 \div 5 = 3\] squares can fit along the width.

So, \(5 \times 3 = 15\) squares can be made from the tag board.

**Check Your Progress**

CANDY Alice is making candy baskets using chocolate hearts and lollipops. She is tying each piece of candy with either a red piece of string or a green piece of string. She has 64 inches of red string and 56 inches of green string. She wants to cut the pieces of string equal lengths and use all of the string she has.

**a.** What is the length of the longest piece of string that can be cut?

8 in.

**b.** How many pieces of string can be cut if the pieces are 8 inches long?

15
Problem-Solving Investigation: Make an Organized List

**EXAMPLE** Make an Organized List

**PASSWORD** In order to log on to the computer at school, Miranda must use a password. The password is 2 characters. The first character is the letter A or B followed by a single numeric digit. How many passwords does Miranda have to choose from?

**UNDERSTAND** You know that the password has 2 characters and that the first character is either the letter A or B. You know that the second character is a numeric digit. You need to know how many passwords can be created.

**PLAN** Make an organized list.

**SOLVE**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>A</th>
<th>B</th>
<th>A</th>
<th>B</th>
<th>A</th>
<th>B</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>A</th>
<th>B</th>
<th>A</th>
<th>B</th>
<th>A</th>
<th>B</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5</td>
<td>6</td>
<td>6</td>
<td>7</td>
<td>7</td>
<td>8</td>
<td>8</td>
<td>9</td>
<td>9</td>
</tr>
</tbody>
</table>

There are 20 passwords.

**CHECK** Draw a tree diagram to check the result.

**Check Your Progress** **DELI** At a deli, customers can choose from ham or turkey on wheat, rye, or multi-grain bread. How many sandwich possibilities are there?
Fractions having the same value are called equivalent fractions.

A fraction is in simplest form when the greatest common factor of the numerator and the denominator is 1.

### Examples

Write each fraction in simplest form.

1. \(\frac{12}{45}\)

To write a fraction in simplest form, you can divide by common factors or divide by the GCF. Let’s divide by the GCF.

First, find the GCF of the numerator and denominator.

Factors of 12: 1, 2, 3, 4, 6, 12

Factors of 45: 1, 3, 5, 9, 15, 45

The GCF of 12 and 45 is 3.

Then, divide the numerator and the denominator by 3.

\[
\frac{12}{45} = \frac{12 \div 3}{45 \div 3} = \frac{4}{15}
\]

So, \(\frac{12}{45}\) written in simplest form is \(\frac{4}{15}\).
factors of 40: 1, 2, 4, 5, 8, 10, 20, 40
factors of 64: 1, 2, 4, 8, 16, 32, 64
The GCF of 40 and 64 is 8.

\[
\frac{40}{64} = \frac{40 \div 8}{64 \div 8} = \frac{5}{8}
\]
So, \(\frac{40}{64}\) written in simplest form is \(\frac{5}{8}\).

Check Your Progress Write each fraction in simplest form.

a. \(\frac{32}{40}\)
b. \(\frac{28}{49}\)

EXAMPLE MUSIC Two notes form a perfect fifth if the simplified fraction of the frequencies of the notes equals \(\frac{3}{4}\). If note D = 294 Hertz and note G = 392 Hertz, do they form a perfect fifth?

\[
\frac{\text{frequency of note D}}{\text{frequency of note G}} = \frac{294}{392} = \frac{1 \times 3 \times \frac{1}{1} \times \frac{1}{1}}{2 \times 2 \times \frac{1}{1} \times \frac{1}{1}} = \frac{1}{4}
\]
The fraction of the frequency of the notes D and G is \(\frac{3}{4}\).
So, the two notes do form a perfect fifth.

Check Your Progress In a bag of 96 marbles, 18 of the marbles are black. Write the fraction of black marbles in simplest form.

\(\frac{3}{16}\)
Fractions and Decimals

EXAMPLES
Use Mental Math

Write each fraction or mixed number as a decimal.

1. \( \frac{9}{10} \)
   
   THINK
   
   \[ \frac{9}{10} = \frac{90}{100} \]
   
   So, \( \frac{9}{10} = 0.9 \).

2. \( 7\frac{3}{5} \)
   
   \[ 7\frac{3}{5} = 7 + \frac{3}{5} \]
   
   Think of it as a sum.
   
   \[ = 7 + 0.6 \]
   
   You know that \( \frac{3}{5} = 0.6 \).
   
   \[ = 7.6 \]
   
   Add mentally.
   
   So, \( 7\frac{3}{5} = 7.6 \).

Check Your Progress
Write each fraction or mixed number as a decimal.

a. \( \frac{7}{25} \)
   
   \[ 0.28 \]

b. \( 9\frac{1}{5} \)
   
   \[ 9.2 \]
EXAMPLE  Use Pencil and Paper or a Calculator

Write $\frac{1}{8}$ as a decimal.

**METHOD 1** Use paper and pencil.

$$
\begin{array}{c}
\text{0.125} \\
\hline
\text{8} | \text{1.000} \\
\downarrow & \text{Divide 1 by 8.} \\
\text{20} \\
\text{16} \\
\text{40} \\
\text{40} \\
\text{0} \\
\end{array}
$$

Division ends when the remainder is 0.

**METHOD 2** Use a calculator.

$$
1 \div 8 \text{ ENTER } 0.125
$$

So, $\frac{1}{8} = 0.125$.

**Check Your Progress** Write each fraction or mixed number as a decimal.

a. $\frac{2}{5}$

0.4

b. $1\frac{7}{20}$

1.35

**Build Your Vocabulary** (pages 77–78)

A **terminating decimal** is a decimal whose digits end.

Repeating decimals have a pattern in the digits that repeats forever.

Bar notation is used to indicate that a number repeats forever by writing a bar over the digits that repeat.
EXAMPLES  Write Fractions as Repeating Decimals

4 Write \( \frac{1}{11} \) as a decimal. (parallels Example 5 in text)

METHOD 1  Use paper and pencil.

\[
\begin{array}{c|c}
\text{Long Division} & \\
\hline
11 & 1.0000 \quad \text{(continued)} \\
0 & \\
100 & \\
-99 & \\
\hline
1 & \\
\end{array}
\]

So, \( \frac{1}{11} = 0.0909... \).

METHOD 2  Use a calculator.

\[1 \div 11 \quad \text{ENTER} \quad 0.0909...\]

Check Your Progress  Write \( 2\frac{5}{11} \) as a decimal.

2.45

EXAMPLE  Use a Power of 10 (parallels Example 6 in text)

5 CEREAL  Jorge read that 0.72 of his favorite cereal was whole-grain wheat. Find what fraction of his cereal, in simplest form, is whole-grain wheat.

\[
0.72 = \frac{72}{100} \quad \text{(The final digit, 2, is in the hundredths place.)}
\]

\[
\frac{72}{100} = \frac{18}{25} \quad \text{(Simplify.)}
\]

So, \( \frac{18}{25} \) of the cereal is whole-grain wheat.

Check Your Progress  EXERCISE  Jeanette ran 0.86 of a mile. What fraction of a mile did she run?

\[
\frac{43}{50}
\]
Main Idea

- Write fractions as percents and percents as fractions.

**Build Your Vocabulary** (pages 77–78)

**A ratio is a comparison** of two numbers by division.

When a ratio compares a number to 100, it can be written as a percent.

**Examples** Write Ratios as Percents

Write each ratio as a percent.

1. **Diana scored 63 goals out of 100 attempts.**
   
   You can represent 63 out of 100 with a model.
   
   \[
   \frac{63}{100} = 63\%
   \]

2. **In a survey, 31.9 out of 100 people on average preferred crunchy peanut butter.**
   
   \[
   \frac{31.9}{100} = 31.9\%
   \]

**Check Your Progress** Write each ratio as a percent.

a. **Alicia sold 34 of the 100 cookies at the bake sale.**
   
   \[
   \frac{34}{100} = 34\%
   \]

b. **On average, 73.4 out of 100 people preferred the chicken instead of the roast beef.**
   
   \[
   \frac{73.4}{100} = 73.4\%
   \]
**EXAMPLE**  **Write a Fraction as a Percent**

5. Write $\frac{16}{25}$ as a percent.

Since $100 \div 25 = 4$, multiply the numerator and denominator by 4.

$$
\frac{16}{25} \times 4 = \frac{64}{100}
$$

So, $\frac{16}{25} = 64\%$.

**Check Your Progress**  **Write $\frac{11}{20}$ as a percent.**

55%

**EXAMPLE**

4. **FISHING** William caught and released 20 trout on his fishing trip. Twelve of them were rainbow trout. What percent of the trout he caught were rainbow trout?

William caught 12 rainbow trout out of 20 trout.

$$\frac{12}{20} = \frac{60}{100}$$

Write an equivalent fraction with a denominator of 100.

$$= 60\%$$

So, $60\%$ of the trout William caught were rainbow trout.

**Check Your Progress**  **READING** Mitchell read 18 out of 25 chapters of a book during his winter vacation. What percent of chapters did he read?

72%
Main Idea

Write percents as decimals and decimals as percents.

Examples

Write Percents as Decimals

1. Write $47.8\%$ as a decimal.
   
   To write a percent as a decimal, you can either first write the percent as a fraction or divide mentally. Let’s divide mentally.
   
   $47.8\% = \frac{47.8}{100}$
   
   Remove the % symbol and divide by 100.
   
   $= 0.478$
   
   Add leading zero.
   
   So, $47.8\% = 0.478$.

2. Population According to the Administration on Aging, about $28\frac{1}{5}\%$ of the population of the United States is 19 years of age or younger. Write $28\frac{1}{5}\%$ as a decimal.
   
   $28\frac{1}{5}\% = 28.2\%$
   
   Write $\frac{1}{5}$ as 0.2.
   
   $= 28.2$
   
   Remove the % symbol and divide by 100.
   
   $= 0.282$
   
   Add leading zero.
   
   So, $28\frac{1}{5}\% = 0.282$.

Check Your Progress

a. Write $83.2\%$ as a decimal.

   $0.832$

b. Amusement Parks A popular amusement park reports that $17\frac{1}{10}\%$ of its visitors will return at least three times during the year. Write $17\frac{1}{10}\%$ as a decimal.

   $0.171$
EXAMPLE  Write Decimals as Percents

Write 0.33 as a percent.

**METHOD 1** Write the decimal as a fraction.

\[ 0.33 = \frac{33}{100} \]

\[ = 33\% \quad \text{Write the fraction as a percent.} \]

**METHOD 2** Multiply mentally.

\[ 0.33 = 33.0 \quad \text{Multiply by 100.} \]

\[ = 33\% \quad \text{Add the } \% \text{ symbol.} \]

So, \( 0.33 = 33\% \).

**Check Your Progress** Write 0.7 as a percent.

70%

EXAMPLE  POPULATION In 1790, about 0.05 of the population of the United States lived in an urban setting. Write 0.05 as a percent.

\[ 0.05 = \frac{5}{100} \quad \text{Definition of decimal} \]

\[ = 5\% \quad \text{Definition of } \% \text{ symbol} \]

**Check Your Progress** In 2000, the population of Illinois had increased by 0.086 from 1990. Write 0.086 as a percent.

8.6%
Main Idea
- Find the least common multiple of two or more numbers.

Build Your Vocabulary (pages 77–78)

A multiple is the product of a number and any whole number.

The least common multiple (LCM) of two or more numbers is the least of their common multiples, excluding zero.

Examples
Find the LCM of 4 and 6.

Method 1 List the nonzero multiples.

Multiples of 4:
4, 8, 12, 16, 20, 24, 28, 32, 36, ...

Multiples of 6:
6, 12, 18, 24, 30, 36, ...

The common multiples are 12, 24, 36, ...

The LCM of 4 and 6 is 12.

Method 2 Use prime factorization.

4 = 2 · 2
6 = 2 · 3

The LCM is 2 · 2 · 3 or 12.
2. Find the LCM of 4 and 15.

Use Method 2. Find the prime factorization of each number.

\[ 4 = 2 \times 2 \text{ or } 2^2 \]
\[ 15 = 3 \times 5 \]

The prime factors of 4 and 15 are **2, 3, and 5**.

The LCM of 4 and 15 is \( 2^2 \times 3 \times 5 \), or **60**.

**Check Your Progress**

Find the LCM of each set of numbers.

a. 8, 12  
   **24**

b. 6, 14  
   **42**

**Example**

**Work** On an assembly line, machine A must be oiled every 18 minutes, machine B every 24 minutes, and machine C every 48 minutes. If all three machines are turned on at the same time, in how many minutes will all three machines need to be oiled at the same time?

First find the LCM of 18, 24, and 48.

\[ 18 = 2 \times 3 \times 3 \text{ or } 2 \times 3^2 \]
\[ 24 = 2 \times 2 \times 2 \times 3 \text{ or } 2^3 \times 3 \]
\[ 48 = 2 \times 2 \times 2 \times 2 \times 3 \text{ or } 2^4 \times 3 \]

The LCM of 18, 24, and 48 is \( 2^4 \times 3^2 \), or **16 \times 9**. So, all three machines will need to be oiled at the same time in **144** minutes.

**Check Your Progress**

**Lights** Brenda put up three different strands of decorative blinking lights. The first strand blinks every 6 seconds while the second strand blinks every 8 seconds. The third strand blinks every 4 seconds. If all strands blink at the same time, in how many seconds will they again blink at the same time?

**24 seconds**
### Main Idea
- Compare and order fractions, decimals, and percents.

### Vocabulary
- **Rational numbers**: Numbers that can be written as fractions and include fractions, terminating and repeating decimals, and integers.
- **Common denominator**: A common multiple of two or more fractions.
- **Least common denominator (LCD)**: The LCM of the denominators.

### Examples
#### Compare Rational Numbers
Replace each \( \_ \_ \text{CE}h \) with <, >, or = to make a true sentence.

1. \(-3\frac{3}{8} \_ \_ \text{CE}h -3\frac{7}{8}\)

Graph each rational number on a number line.

Mark off equal size increments of \( \frac{1}{8} \) between \(-4 \) and \(-3\).

The number line shows that \(-3\frac{3}{8} > -3\frac{7}{8}\).

2. \(\frac{5}{12} \_ \_ \frac{7}{16}\)

The LCD of the denominators, 12 and 16, is 48.

\[
\frac{5}{12} = \frac{5 \cdot 4}{12 \cdot 4} = \frac{20}{48}
\]

\[
\frac{7}{16} = \frac{7 \cdot 3}{16 \cdot 3} = \frac{21}{48}
\]

Since \(\frac{20}{48} < \frac{21}{48}\), then \(\frac{5}{12} < \frac{7}{16}\).
Check Your Progress
Replace each • with <, >, or = to make a true sentence.

a. \(-2\frac{4}{5} \quad • \quad -2\frac{3}{5}\)

b. \(\frac{5}{8} \quad • \quad \frac{7}{12}\)

EXAMPLE
According to the Pet Food Manufacturer's Association, 11 out of 25 people own large dogs and 13 out of 50 people own medium dogs. Do more people own large or medium dogs?

Write \(\frac{11}{25}\) and \(\frac{13}{50}\) as decimals and compare.

\[
\frac{11}{25} = 0.44 \quad \frac{13}{50} = 0.26
\]

Since 0.44 > 0.26, \(\frac{11}{25} > \frac{13}{50}\).

So, a greater fraction of people own large dogs than own medium dogs.

Check Your Progress
A survey showed that 21 out of 50 people stated that summer is their favorite season and 13 out of 25 people prefer fall. Do more people prefer summer or fall?
BRINGING IT ALL TOGETHER

STUDY GUIDE

4-1
Prime Factorization

Underline the correct terms to complete each sentence.

1. A factor tree is complete when all of the factors at the bottom of the factor tree are (prime, composite) factors.

2. The order of the factors in prime factorization (does, does not) matter.

Find the prime factorization of each number.

3. 36
   \[2 \cdot 2 \cdot 3 \cdot 3\]

4. 48
   \[2 \cdot 2 \cdot 2 \cdot 2 \cdot 3\]

5. 250
   \[5 \cdot 5 \cdot 5 \cdot 2\]

6. 60
   \[2 \cdot 2 \cdot 3 \cdot 5\]

4-2
Greatest Common Factor

Complete each sentence.

7. A Venn diagram shows how elements of sets of numbers are related.

8. A prime factor is a factor that is a prime number.
9. You can find the **GCF** of two numbers by **multiplying** the common prime factors.

Find the common prime factors and GCF of each set of numbers.

10. 20, 24  
    2 and 2; 4

11. 28, 42  
    2 and 7; 14

4-3

**Problem-Solving Investigation: Make an Organized List**

12. **CLOTHES** Lucas has a pair of brown pants and a pair of black pants. He has a white dress shirt, a blue dress shirt, and a tan dress shirt. He has a striped tie and a polka-dotted tie. Assuming he can wear any combination, how many combinations of one pair of pants, one dress shirt, and one tie can Lucas wear?

   12

4-4

**Simplifying Fractions**

Complete the sentence.

13. To find the simplest form of a fraction, **divide** the numerator and the denominator by the **GCF**.

Write each fraction in simplest form.

14. \( \frac{18}{24} \)  
    \( \frac{3}{4} \)

15. \( \frac{15}{60} \)  
    \( \frac{1}{4} \)

4-5

**Fractions and Decimals**

Write each fraction or mixed number as a decimal. Use bar notation if the decimal is a repeating decimal.

16. \( 3 \frac{2}{3} \)  
    3.6

17. \( 5 \frac{3}{4} \)  
    5.75

18. \( \frac{2}{5} \)  
    0.4

19. \( 7 \frac{3}{8} \)  
    7.375

20. \( 6 \frac{1}{2} \)  
    6.5

21. \( \frac{7}{10} \)  
    0.7
22. Write the ratio that compares 4 to 25 in three different ways.

4 out of 25; \( \frac{4}{25} \); 4:25

23. Write the ratio in exercise 22 as a percent. \( 16\% \)

24. Write 88\% as a fraction in simplest form. \( \frac{22}{25} \)

25. Write \( \frac{9}{20} \) as a percent. \( 45\% \)

26. 69\% \( 0.69 \)

27. 3\% \( 0.03 \)

28. \( 32\frac{1}{4}\% \) \( 0.3425 \)

29. 0.47 \( 47\% \)

30. 0.5775 \( 57\frac{3}{4}\% \)

31. 0.09 \( 9\% \)

32. 15, 36 \( 180 \)

33. 21, 70 \( 210 \)

34. 16, 20 \( 80 \)

35. 6, 9, 24 \( 72 \)

36. 12, 18, 30 \( 180 \)

37. 14, 28, 35 \( 140 \)

38. \( \frac{14}{35} \) \( \frac{12}{20} \) \(<\) 

39. \( \frac{21}{49} \) \( \frac{18}{63} \) \( >\)
ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
   • You are probably ready for the Chapter Test.
   • You may want to take the Chapter 4 Practice Test on page 225 of your textbook as a final check.

☐ I used my Foldables or Study Notebook to complete the review of all or most lessons.
   • You should complete the Chapter 4 Study Guide and Review on pages 221–224 of your textbook.
   • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
   • You may want to take the Chapter 4 Practice Test on page 225 of your textbook.

☐ I asked for help from someone else to complete the review of all or most lessons.
   • You should review the examples and concepts in your Study Notebook and Chapter 4 Foldables.
   • Then complete the Chapter 1 Study Guide and Review on pages 221–224 of your textbook.
   • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
   • You may also want to take the Chapter 4 Practice Test on page 225 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature

Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 4.
Applying Fractions

Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with a sheet of 11" by 17" paper, four index cards, and glue.

**STEP 1** Fold the paper in half widthwise.

**STEP 2** Open and fold along the length about \(2\frac{1}{2}\)" from the bottom.

**STEP 3** Glue the edges on either side to form two pockets.

**STEP 4** Label the pockets *Fractions* and *Mixed Numbers*, respectively. Place two index cards in each pocket.

**NOTE-TAKING TIP:** When you take notes, place a question mark next to any concepts you do not understand. Be sure to ask your teacher to clarify these concepts before a test.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 5. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>compatible numbers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>like fractions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>multiplicative inverse</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[MUHL-tuh-PLIH-kuh-tihv]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>reciprocal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[rih-SIH-pruh-kuhl]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>unlike fractions</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Estimating with Fractions

**EXAMPLES** Estimate with Mixed Numbers

Estimate.

1. \(5\frac{1}{4} + 3\frac{5}{8}\)

\[
5\frac{1}{4} + 3\frac{5}{8} \rightarrow 5 + \frac{4}{4} = 9
\]

The sum is about 9.

2. \(7\frac{3}{4} \times 1\frac{7}{8}\)

\[
7\frac{3}{4} \times 1\frac{7}{8} \rightarrow 8 \times 2 = 16
\]

The sum is about 16.

**Check Your Progress** Estimate.

\(\text{a. } 2\frac{7}{9} + 5\frac{1}{4}\)  

\(\text{b. } 4\frac{2}{3} \times 3\frac{1}{8}\)

- 3 + 5 or 8
- 5 \times 3 or 15

**EXAMPLES** Estimate with Fractions

Estimate.

\(\frac{1}{3} + \frac{4}{7}\)

\[
\frac{1}{3} \text{ is about } \frac{1}{2}.
\]

\[
\frac{4}{7} \text{ is about } \frac{1}{2}.
\]

\[
\frac{1}{3} + \frac{4}{7} \rightarrow \frac{1}{2} + \frac{1}{2} = 1
\]

The sum is about 1.

-记录主要想法、定义和其他关于估算分数的笔记在学习卡片上。将这些卡片放在您 Foldable 的“分数”口袋中。

Math Connects, Course 2
**Remember It**

Some fractions are easy to round because they are close to 1. Examples of these kinds of fractions are ones where the numerator is one less than the denominator, such as \( \frac{4}{5} \) or \( \frac{7}{8} \).

\[ \frac{5}{8} - \frac{1}{4} \]

\[ \frac{5}{8} - \frac{1}{4} \rightarrow \frac{1}{2} - 0 = \frac{1}{2} \]

The difference is about \( \frac{1}{2} \).

\[ \frac{5}{6} \div \frac{4}{5} \]

\[ \frac{5}{6} \div \frac{3}{4} \approx \frac{1}{1} = 1 \]

\[ \frac{5}{6} \approx 1 \text{ and } \frac{3}{4} \approx 1. \]

**Check Your Progress**

Estimate.

a. \( \frac{8}{9} + \frac{1}{6} \)

b. \( \frac{11}{12} - \frac{2}{9} \)

c. \( \frac{3}{5} \div \frac{7}{8} \)

\[ 1 + 0 \text{ or } 1 \]

\[ 1 - 0 \text{ or } 1 \]

\[ \frac{1}{2} \div 1 \text{ or } \frac{1}{2} \]

**Build Your Vocabulary**

Numbers that are easy to compute mentally are called compatible numbers.

**Example**

Use Compatible Numbers

Estimate \( \frac{3}{4} \times 21 \) using compatible numbers.

\[ \frac{3}{4} \times 21 \approx \frac{3}{4} \times 20 \text{ or } 15 \]

Round 21 to 20, since 20 is divisible by 4.

**Check Your Progress**

Estimate \( \frac{2}{3} \times 17 \) using compatible numbers.

\[ 12 \]
EXAMPLES Add and Subtract Like Fractions

Add or subtract. Write in simplest form.

\[ \frac{7}{12} + \frac{4}{12} = \frac{7 + 4}{12} \]

Add the \( \text{numerators} \).

\[ = \frac{11}{12} \]

Write the sum over the \( \text{denominator} \).

\[ \frac{5}{6} - \frac{1}{6} = \frac{5 - 1}{6} \]

Subtract the \( \text{numerators} \).

\[ = \frac{4}{6} \text{ or } \frac{2}{3} \]

Write the difference over the \( \text{denominator} \). Simplify.

EXAMPLES Add and Subtract Unlike Fractions

Add or subtract. Write in simplest form.

\[ \frac{1}{3} + \frac{1}{9} \]

To add or subtract unlike fractions, you can use a \( \text{model} \) or the \( \text{LCD} \). Let’s use the LCD.

The least common denominator of 3 and 9 is 9.

\[ \frac{1}{3} = \frac{1 \times 3}{3 \times 3} = \frac{3}{9} \]

Rename \( \frac{1}{3} \) using the \( \text{LCD} \).

\[ \frac{1}{3} \quad \rightarrow \quad \frac{3}{9} \]

\[ + \frac{1}{9} \quad \rightarrow \quad + \frac{1}{9} \]

\[ \frac{4}{9} \]

So, \( \frac{1}{3} + \frac{1}{9} = \frac{4}{9} \).
The LCD of 4 and 6 is 12.

\[ \frac{3}{4} - \frac{1}{6} \]

Rename each fraction using the LCD.

\[ \frac{3 \times 3}{4 \times 3} \rightarrow \frac{9}{12} \]

\[ \frac{1 \times 2}{6 \times 2} \rightarrow -\frac{2}{12} \]

So, \( \frac{3}{4} - \frac{1}{6} = \frac{7}{12} \).

**Check Your Progress**

Add or subtract. Write in simplest form.

a. \( \frac{7}{15} + \frac{4}{15} \)

b. \( \frac{3}{8} + \frac{1}{4} \)

c. \( \frac{7}{9} - \frac{1}{6} \)

- \( \frac{11}{15} \)
- \( \frac{5}{8} \)
- \( \frac{11}{18} \)

**EXAMPLE**

**ART** A picture mounted on art board is \( \frac{1}{8} \) inch thick. The frame for the picture is \( \frac{1}{2} \) inch thick. How much thicker than the picture is the frame?

The phrase *how much thicker* suggests subtraction, so find \( \frac{1}{2} - \frac{1}{8} \).

\[ \frac{1}{2} - \frac{1}{8} = \frac{4}{8} - \frac{1}{8} \]

 Rename the fractions using the LCD.

\[ = \frac{3}{8} \]

Subtract the numerators.

The frame is \( \frac{3}{8} \) inch thicker than the picture.

**Check Your Progress** **RUNNING** Gregory ran \( \frac{3}{4} \) of a mile on Monday and \( \frac{5}{6} \) of a mile on Tuesday. How much more of a mile did he run on Tuesday?

\[ \frac{1}{12} \]
**Main Idea**

Add and subtract mixed numbers.

**Examples**

**Add and Subtract Mixed Numbers**

Add or subtract. Write in simplest form.

1. \( \frac{3}{12} + \frac{14}{12} \)

   **Estimate** \( 3 + 15 = 18 \)

   \[
   \frac{3}{12} + \frac{14}{12} = \frac{17}{12} \]
   
   Simplify. Compare the sum to the estimate.

2. \( \frac{9}{10} - \frac{4}{5} \)

   **Estimate** \( 10 - 5 = 5 \)

   \[
   \frac{9}{10} - \frac{4}{5} = \frac{5}{10} \]
   
   Rename the fraction using the LCD. Simplify. Compare the sum to the estimate.

**Examples**

**Rename Mixed Numbers to Subtract**

Subtract. Write in simplest form.

1. \( \frac{8}{5} - \frac{3}{5} \)

   Rename \( \frac{8}{5} \) as \( \frac{7}{5} \).

   \[
   \frac{8}{5} - \frac{3}{5} = \frac{4}{5} \]
   
   First subtract the whole numbers and then the fractions.
Subtract.

\[ \begin{align*}
11 & \quad \rightarrow \quad 10\frac{2}{3} \\
-8\frac{2}{3} & \quad \rightarrow \quad -8\frac{2}{3} \\
\hline
2\frac{1}{3} & \quad \text{Subtract.}
\end{align*} \]

**Check Your Progress** Add or subtract. Write in simplest form.

a. \( \frac{5}{14} + \frac{4}{14} \)  

b. \( 6\frac{2}{9} - 3\frac{5}{9} \)  
c. \( 9\frac{3}{8} - 5\frac{3}{4} \)

**EXAMPLE**

**COOKING** A quiche recipe calls for \( 2\frac{3}{4} \) cups of grated cheese. A recipe for quesadillas requires \( 1\frac{1}{3} \) cups of grated cheese. What is the total amount of grated cheese needed for both recipes?

\[
\begin{align*}
2\frac{3}{4} + 1\frac{1}{3} & = 2\frac{9}{12} + 1\frac{4}{12} \\
= & = 3 + 1\frac{1}{12} \quad \text{or} \quad 4\frac{1}{12} \\
\end{align*}
\]

The total amount of grated cheese needed is \( 4\frac{1}{12} \) cups.

**Check Your Progress** TIME Jordan spent \( 3\frac{1}{6} \) hours at the mall and \( 2\frac{1}{4} \) hours at the movies. How many more hours did he spend at the mall than at the movies?

\( \frac{11}{12} \) h
EXAMPLE  Eliminate Possibilities

**GAMES**  On a television game show, the winning contestant must answer three questions correctly to win the grand prize. Each question is worth twice as many points as the question before it. The third question is worth 1,000 points. How much is the first question worth—250, 500, or 2,000 points?

**UNDERSTAND**  You know that there are three questions and each question is worth twice as many points as the question before it. You know that the third question is worth 1,000 points.

**PLAN**  Eliminate answers that are not reasonable.

**SOLVE**  The first question cannot be worth 2,000 points since each question after it would have to be worth more than 2,000 points, and the third question is only 1,000 points. So, eliminate that choice. If the first question is worth 500 points, then the second question would be worth 1,000 points and the third question would be worth 2,000 points. So, eliminate that choice. The reasonable answer is 250 points.

**CHECK**  If the first question is worth 250 points, then the second question would be worth 500 points, and the third question would be worth 1,000 points. So, the answer is correct.

**Check Your Progress**  **CELL PHONES**  A cell phone company charges $35 for 500 free minutes and $0.50 for each additional minute. Using this plan, what is a reasonable price a customer would pay for using 524 minutes—$32, $40, or $47?

$47
EXAMPLES

Multiply Fractions

Multiply. Write in simplest form.

1. \(\frac{1}{8} \times \frac{1}{9}\)
   \[\frac{1}{8} \times \frac{1}{9} = \frac{1 \times 1}{8 \times 9}\]
   \[= \frac{1}{72}\]
   Multiply the numerators.
   Multiply the denominators.
   Simplify.

2. \(6 \times \frac{1}{3}\)
   \[6 \times \frac{1}{3} = \frac{6}{1} \times \frac{1}{3}\]
   Write 6 as \(\frac{6}{1}\).
   Multiply the numerators and the denominators.
   \[= \frac{6 \times 1}{1 \times 3}\]
   \[= \frac{6}{3}\] or \(2\)
   Simplify.

Check Your Progress

Multiply. Write in simplest form.

a. \(\frac{1}{5} \times \frac{1}{7}\)
   \[\frac{1}{35}\]

b. \(12 \times \frac{1}{6}\)
   \[2\]

EXAMPLE

Simplify Before Multiplying

Multiply. Write in simplest form.

3. \(\frac{3}{12} \times \frac{4}{5}\)
   \[\frac{3}{12} \times \frac{4}{5} = \frac{3}{12} \times \frac{4}{5}\]
   Divide 4 and 12 by their GCF, 4.
   \[\frac{3 \times 1}{3 \times 5}\]
   Multiply the numerators and the denominators.
   \[= \frac{1}{5}\]
   Simplify.
**EXAMPLE** Multiply Mixed Numbers

Multiply \( \frac{1}{3} \times 6\frac{6}{7} \). Write in simplest form.

**METHOD 1** Rename the mixed number.

\[
\frac{1}{3} \times 6\frac{6}{7} = \frac{1}{3} \times \frac{48}{7}
\]

Rename \( 6\frac{6}{7} \) as an improper fraction, \( \frac{48}{7} \).

\[
= \frac{1 \times 16}{1 \times 7}
\]

Multiply.

\[
= \frac{16}{7} \quad \text{or} \quad 2\frac{2}{7}
\]

Simplify.

**METHOD 2** Use mental math.

\[
\frac{1}{3} \times 6\frac{6}{7} = \frac{1}{3} \times \left( 6 + \frac{6}{7} \right)
\]

Write \( 6\frac{6}{7} \) as a sum of its parts.

\[
= \left( \frac{1}{3} \times 6 \right) + \left( \frac{1}{3} \times \frac{6}{7} \right)
\]

Distributive Property

\[
= 2 + \frac{2}{7} \quad \text{or} \quad 2\frac{2}{7}
\]

Multiply.

**Check Your Progress** Multiply. Write in simplest form.

a. \( \frac{4}{9} \times \frac{6}{7} \)

b. \( \frac{1}{6} \times 4\frac{6}{9} \)
**Main Idea**

- Solve equations with rational number solutions.

**Build Your Vocabulary** (page 103)

Two numbers whose **product** is **1** are called **multiplicative inverses**.

Reciprocals is another name given to **multiplicative inverses**.

**Examples**

**Find Multiplicative Inverses**

Find the multiplicative inverse of each number.

1. \( \frac{4}{7} \)

\[
\frac{4}{7} \cdot \frac{7}{4} = 1
\]

Multiply \( \frac{4}{7} \) by \( \frac{7}{4} \) to get the product **1**.

The multiplicative inverse of \( \frac{4}{7} \) is \( \frac{7}{4} \), or \( 1 \frac{3}{4} \).

2. \( 6 \frac{1}{4} \)

\[
6 \frac{1}{4} = \frac{25}{4}
\]

 Rename the **mixed number** as an improper fraction.

\[
\frac{25}{4} \cdot \frac{4}{25} = 1
\]

Multiply \( \frac{25}{4} \) by \( \frac{4}{25} \) to get the product **1**.

The multiplicative inverse of \( 6 \frac{1}{4} \) is \( \frac{4}{25} \).

**Check Your Progress**

Find the multiplicative inverse of each number.

a. \( \frac{5}{8} \)  

\[
\frac{8}{5} \text{ or } 1 \frac{3}{5}
\]

b. \( 4 \frac{1}{3} \)  

\[
\frac{3}{13}
\]
**EXAMPLE** Solve a Division Equation

Solve $11 = \frac{p}{6}$. Check your solution.

1. Write the equation.
   
   $$11 = \frac{p}{6}$$

2. Multiply each side by 6.
   
   $$11 \cdot 6 = \frac{p}{6} \cdot 6$$

   
   $$66 = p$$

**Check**

1. Write the original equation.
   
   $$11 = \frac{p}{6}$$

2. Replace $p$ with 66.
   
   $$11 = \frac{66}{6}$$

   
   $$11 = 11$$

The solution is 66.

**EXAMPLE** Use a Reciprocal to Solve an Equation

Solve $\frac{2}{5}x = \frac{6}{15}$. (parallels Example 5 in text)

1. Write the equation.
   
   $$\frac{2}{5}x = \frac{6}{15}$$

2. Multiply each side by the reciprocal of $\frac{2}{5}$.
   
   $$\left(\frac{5}{2}\right) \frac{2}{5}x = \left(\frac{5}{2}\right) \left(\frac{6}{15}\right)$$

   
   $$x = \frac{30}{30} = 1$$

**Check Your Progress** Solve.

a. $\frac{m}{9} = 4$

   $$m = 36$$

b. $\frac{3}{8}x = \frac{3}{4}$

   $$x = 2$$
5–7  Dividing Fractions and Mixed Numbers

**Main Idea**
- Divide fractions and mixed numbers.

**Key Concept**
**Division by a Fraction**
To divide by a fraction, multiply by its multiplicative inverse or reciprocal.

**Write It**
Will the quotient \(7 \frac{1}{6} \div 3 \frac{2}{3}\) be a fraction less than 1 or greater than 1? Explain.

**Example 1**
**Divide by a Fraction**

Find \(\frac{2}{3} \div \frac{4}{9}\). Write in simplest form.

\[
\frac{2}{3} \div \frac{4}{9} = \frac{2}{3} \cdot \frac{9}{4}
\]

Multiply by the reciprocal \(\frac{9}{4}\).

\[
= \frac{2}{3} \cdot \frac{9}{4} = \frac{2 \cdot 9}{3 \cdot 4} = \frac{18}{12} = \frac{3}{2}
\]

Divide out common factors.

\[
= \frac{3}{2} \text{ or } 1 \frac{1}{2}
\]

Multiply and simplify.

**Example 2**
**Divide by Mixed Numbers**

Find \(\frac{5}{6} \div 2 \frac{1}{2}\). Write in simplest form.

**Estimate**
\[1 \div \frac{5}{2} = 1 \times \frac{2}{5} \text{ or } \frac{2}{5}\]

\[
\frac{5}{6} \div 2 \frac{1}{2} = \frac{5}{6} \div \frac{5}{2}
\]

Rename \(2 \frac{1}{2}\) as an improper fraction.

\[
= \frac{5}{6} \cdot \frac{2}{5}
\]

Multiply by the reciprocal of \(\frac{5}{2}\).

\[
= \frac{5}{6} \cdot \frac{2}{5} = \frac{5 \cdot 2}{6 \cdot 5} = \frac{1}{3}
\]

Divide out common factors.

\[
= \frac{1}{3}
\]

Multiply. The quotient is close to the estimate.

**Check Your Progress**
Divide. Write in simplest form.

a. \(\frac{6}{7} \div \frac{2}{5}\)

\[
= 2 \frac{1}{7}
\]

b. \(\frac{3}{8} \div 2 \frac{1}{2}\)

\[
= \frac{3}{20}
\]

Math Connects, Course 2 115
**EXAMPLE**

A bottling machine needs to be restocked with new lids every $2\frac{3}{4}$ hours. If the machine runs $19\frac{1}{4}$ hours, how many times will it have to be restocked with lids?

$$19\frac{1}{4} \div 2\frac{3}{4} = \frac{77}{4} \div \frac{11}{4}$$

Rename the mixed numbers as improper fractions.

$$= \frac{77}{4} \cdot \frac{4}{11}$$

Multiply by the reciprocal of $\frac{11}{4}$, which is $\frac{4}{11}$.

$$= \frac{77}{4} \cdot \frac{1}{11}$$

Divide out common factors.

$$= \frac{7}{1} \text{ or } 7$$

Multiply.

So, the machine will need to restocked 7 times.

**Check Your Progress**

A rectangular table is $5\frac{5}{6}$ feet long. If the area of the table is $20\frac{5}{12}$ square feet, how wide is the table?

$3\frac{1}{2}$ ft


**5-1**

**Estimating with Fractions**

Estimate.

1. \(\frac{8}{3} + \frac{7}{4}\) [16]
2. \(\frac{11}{8} \div \frac{3}{6}\) [3]

**5-2**

**Adding and Subtracting Fractions**

Add or subtract. Write in simplest form.

3. \(\frac{7}{8} + \frac{3}{8}\) \(1\frac{1}{4}\)
4. \(\frac{5}{6} - \frac{1}{3}\) \(\frac{1}{2}\)
5. \(\frac{1}{5} + \frac{3}{4}\) \(\frac{19}{20}\)

**5-3**

**Adding and Subtracting Mixed Numbers**

Add or subtract. Write in simplest form.

6. \(3\frac{7}{8} + 6\frac{1}{4}\) \(10\frac{1}{8}\)
7. \(7\frac{1}{6} + 2\frac{5}{12}\) \(9\frac{7}{12}\)
8. \(8\frac{3}{7} - 4\frac{5}{7}\) \(3\frac{5}{7}\)
9. \(9\frac{2}{9} - 1\frac{2}{3}\) \(7\frac{5}{9}\)

**VOCABULARY PUZZLEMAKER**

To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 5, go to:

glencoe.com

**BUILD YOUR VOCABULARY**

You can use your completed Vocabulary Builder (page 103) to help you solve the puzzle.

**FOLDABLES**

Use your Chapter 5 Foldable to help you study for your chapter test.
10. **READING** Joel read $\frac{5}{8}$ of a novel. If the novel has 600 pages, is 250, 300, or 375 a reasonable number of pages that Joel has read?  

375

### 5-5 Multiplying Fractions and Mixed Numbers

Multiply. Write in simplest form.

11. $\frac{2}{7} \times 4\frac{1}{5}$

12. $\frac{1}{6} \times 3\frac{1}{4}$

13. $5\frac{1}{6} \times \frac{2}{3}$

14. $\frac{5}{8} \times 4\frac{4}{5}$

### 5-6 Algebra: Solving Equations

Find the multiplicative inverse of each number.

15. $\frac{3}{5} \times \frac{5}{3}$

16. $1\frac{1}{2} \times \frac{2}{3}$

17. $3 \times \frac{1}{3}$

Solve each equation.

18. $\frac{1}{3}a = \frac{5}{6}$

19. $-4 = \frac{k}{3}$

### 5-7 Dividing Fractions and Mixed Numbers

Divide. Write in simplest form.

20. $\frac{1}{4} \div \frac{2}{3}$

21. $\frac{7}{8} \div \frac{2}{3}$

22. $6 \div 1\frac{1}{3}$

23. $5\frac{3}{4} \div 2\frac{1}{2}$
ARE YOU READY FOR
THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
   - You are probably ready for the Chapter Test.
   - You may want to take the Chapter 5 Practice Test on page 275 of your textbook as a final check.

☐ I used my Foldable or Study Notebook to complete the review of all or most lessons.
   - You should complete the Chapter 5 Study Guide and Review on pages 271–274 of your textbook.
   - If you are unsure of any concepts or skills, refer back to the specific lesson(s).
   - You may also want to take the Chapter 5 Practice Test on page 275 of your textbook.

☐ I asked for help from someone else to complete the review of all or most lessons.
   - You should review the examples and concepts in your Study Notebook and Chapter 5 Foldable.
   - Then complete the Chapter 5 Study Guide and Review on pages 271–274 of your textbook.
   - If you are unsure of any concepts or skills, refer back to the specific lesson(s).
   - You may also want to take the Chapter 5 Practice Test on page 275 of your textbook.

Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 5.

Student Signature  Parent/Guardian Signature
Teacher Signature
Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

**Begin with a sheet of notebook paper.**

**STEP 1** Fold lengthwise to the holes.

**STEP 2** Cut along the top line and then make equal cuts to form 7 tabs.

**STEP 3** Label the major topics as shown.

**NOTE-TAKING TIP:** When you take notes, it may be helpful to include an example for each term or concept learned.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 6. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>cross products</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>equivalent ratios</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>gram</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>kilogram</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>liter</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>meter</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>metric system</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>proportion</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>proportional</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition</td>
<td>Description or Example</td>
</tr>
<tr>
<td>-------------------------</td>
<td>---------------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>ratio</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>scale</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>scale drawing</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>scale factor</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>scale model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>slope</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>unit rate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>unit ratio</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Main Idea**

- Write ratios as fractions in simplest form and determine whether two ratios are equivalent.

**BUILD YOUR VOCABULARY** (pages 121–122)

A **ratio** is a comparison of two quantities by division.

Ratios that express the **same** relationship between two quantities are **equivalent ratios**.

**Example**

**Write Ratios in Simplest Form**

1. **Apples** Mr. Gale bought a basket of apples. Using the table, write a ratio comparing the Red Delicious apples to the Granny Smith apples as a fraction in simplest form.

<table>
<thead>
<tr>
<th>Mr. Gale’s Apples</th>
<th>12 Fuji</th>
<th>9 Granny Smith</th>
<th>30 Red Delicious</th>
</tr>
</thead>
</table>

Red Delicious: Granny Smith \( \frac{30}{9} = \frac{10}{3} \) or \( \frac{10}{3} \)

The ratio of Red Delicious apples to Granny Smith apples is \( \frac{10}{3} \).

**Example**

**Identify Equivalent Ratios**

2. Determine whether the ratios 12 onions to 15 potatoes and 32 onions to 40 potatoes are equivalent.

12 onions : 15 potatoes = \( \frac{12}{15} = \frac{3}{5} \) or \( \frac{4}{5} \)

32 onions : 40 potatoes = \( \frac{32}{40} = \frac{8}{10} = \frac{4}{5} \) or \( \frac{4}{5} \)

The ratios simplify to the same fraction. They are **equivalent**.

**Organize It**

Record a term or concept from Lesson 6–1 under the Ratios tab and write a definition along with an example to the right of the definition.

**Foldables**
Check Your Progress

a. **FLOWERS** A garden has 18 roses and 24 tulips. Write a ratio comparing roses to tulips as a fraction in simplest form.

\[
\frac{3}{4}
\]

b. Determine whether the ratios 3 cups vinegar to 8 cups water and 5 cups vinegar to 12 cups water are equivalent.

No

**EXAMPLE**

**POOLS** It is recommended that no more than one person be allowed into the shallow end of an outdoor public pool for every 15 square feet of surface area. If a local pool’s shallow end has a surface area of 1,800 square feet, are the lifeguards correct to allow 120 people into that part of the pool?

**Recommended Ratio**

\[1:15 = \frac{1}{15}\] persons per square feet

**Actual Ratio**

\[
\frac{120}{1,800} = \frac{120}{1,800} \text{ or } \frac{1}{15}\] persons per square feet

Since the ratios simplify to the same fraction, they are equivalent. The lifeguards are correct.

**Check Your Progress** **SCHOOL** A district claims that they have 1 teacher for every 15 students. If they actually have 2,700 students and 135 teachers, is their claim correct?

No
Build Your Vocabulary (pages 121–122)

A ratio that compares two quantities with different kinds of units is called a rate.

When a rate is simplified so that it has a denominator of 1 unit, it is called a unit rate.

Examples

Find Unit Rates

1. **READING** Julia read 52 pages in 2 hours. What is the average number of pages she read per hour?

   Write the rate as a fraction. Then find an equivalent rate with a denominator of 1.

   \[
   \text{52 pages in 2 hours} = \frac{52 \text{ pages}}{2 \text{ hours}}
   \]

   Write the rate as a fraction.

   \[
   = \frac{52 \text{ pages}}{2 \text{ hours}} \times \frac{2}{2}
   \]

   Divide the numerator and denominator by 2.

   \[
   = \frac{26 \text{ pages}}{1 \text{ hour}}
   \]

   Simplify.

2. **SODA** Find the unit price per can if it costs $3 for 6 cans of soda. Round to the nearest hundredth if necessary.

   \[
   \text{$3 for 6 cans} = \frac{$3}{6 \text{ cans}}
   \]

   Write the rate as a fraction.

   \[
   = \frac{$3 \div 6}{6 \text{ cans} \div 6}
   \]

   Divide the numerator and the denominator by 6.

   \[
   = \frac{$0.50}{1 \text{ can}}
   \]

   Simplify.
Check Your Progress

Find each unit rate.

a. 16 laps in 4 minutes
   \[ \frac{16 \text{ laps}}{4 \text{ minutes}} = \frac{4 \text{ laps}}{1 \text{ minute}} \]

b. $3 for one dozen cookies
   \[ \frac{\$3}{1 \text{ dozen}} = \frac{\$0.25}{1 \text{ cookie}} \]

Example

Compare Using Unit Rates

TEST EXAMPLE

The costs of 4 different sizes of orange juice are shown in the table. Which container costs the least per ounce?

<table>
<thead>
<tr>
<th>Amount</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 oz</td>
<td>$1.28</td>
</tr>
<tr>
<td>32 oz</td>
<td>$1.92</td>
</tr>
<tr>
<td>64 oz</td>
<td>$2.56</td>
</tr>
<tr>
<td>96 oz</td>
<td>$3.36</td>
</tr>
</tbody>
</table>

A 96-oz container
B 64-oz container
C 32-oz container
D 16-oz container

Read the Item

Find the unit price, or the cost per ounce of each size of orange juice. Divide the price by the number of ounces.

Solve the Item

\[ \frac{\$1.28}{16 \text{ ounces}} = \frac{\$0.08}{1 \text{ ounce}} \]
\[ \frac{\$1.92}{32 \text{ ounces}} = \frac{\$0.06}{1 \text{ ounce}} \]
\[ \frac{\$2.56}{64 \text{ ounces}} = \frac{\$0.04}{1 \text{ ounce}} \]
\[ \frac{\$3.36}{96 \text{ ounces}} = \frac{\$0.035}{1 \text{ ounce}} \]

The 96-ounce container of orange juice costs the least per ounce. The answer is A.
Check Your Progress

MULTIPLE CHOICE The costs of different sizes of bottles of laundry detergent are shown below. Which bottle costs the least per ounce?

A 96-oz container
B 64-oz container
C 32-oz container
D 16-oz container

Use a Unit Rate

POTATOES An assistant cook peeled 18 potatoes in 6 minutes. At this rate, how many potatoes can he peel in 50 minutes?

Find the unit rate.

18 potatoes in 6 minutes = \( \frac{18}{6} = \frac{3}{1} \)

The unit rate is 3 potatoes per minute.

\( \frac{3 \text{ potatoes}}{1 \text{ min}} \cdot 50 \text{ min} = 150 \text{ potatoes} \)

He can peel 150 potatoes in 50 minutes.

Check Your Progress Sarah can paint 21 beads in 7 minutes. At this rate, how many beads can she paint in one hour?
Main Idea

Identify rate of change and slope using tables and graphs.

Build Your Vocabulary (pages 121–122)

A rate of change is a rate that describes how one quantity changes in relation to another and is usually expressed as a unit rate.

Example

Find Rate of Change from a Table

The table shows the number of miles a car drove on a trip. Use the information to find the approximate rate of change.

<table>
<thead>
<tr>
<th>Distance (miles)</th>
<th>65</th>
<th>130</th>
<th>195</th>
<th>260</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (hours)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

\[
\frac{\text{change in distance}}{\text{change in time}} = \frac{65}{1}
\]

The distance increased 65 miles for every hour.

So, the rate was 65 miles per hour.

Check Your Progress

The table shows the number of miles a car drove on a trip. Use the information to find the rate of change.

<table>
<thead>
<tr>
<th>Distance (miles)</th>
<th>44</th>
<th>88</th>
<th>132</th>
<th>176</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel (gallons)</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

22 miles per gallon
The constant rate of change in y with respect to the constant change in x is called the **slope** of a line.

**EXAMPLE**

**Find Rate of Change from a Graph**

**GRAPH THE DATA** Find the slope of the line. Explain what the slope represents. (parallels Example 3 in text)

Graph the points and connect them with a line.

<table>
<thead>
<tr>
<th>Hours</th>
<th>Amount Earned</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$45</td>
</tr>
<tr>
<td>6</td>
<td>$90</td>
</tr>
<tr>
<td>9</td>
<td>$135</td>
</tr>
</tbody>
</table>

Pick two points on the line, such as (3, 45) and (6, 90), to find the slope.

\[
\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{90 - 45}{6 - 3} = \frac{45}{3} = 15
\]

The slope is \(15\) and represents the amount earned per hour.

**Check Your Progress**

The table shows the cost of renting a bicycle. Graph the data. Find the slope of the line. Explain what the slope represents.

<table>
<thead>
<tr>
<th>Hours</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$8</td>
</tr>
<tr>
<td>4</td>
<td>$16</td>
</tr>
<tr>
<td>6</td>
<td>$24</td>
</tr>
</tbody>
</table>

The slope is $4 and represents the cost per hour to rent a bicycle.
**MAIN IDEA**

- Change units in the customary system.

**BUILD YOUR VOCABULARY** (pages 121–122)

A **unit ratio** is a ratio in which the denominator is 1 unit.

**EXAMPLES**

**Convert Larger Units to Smaller Units**

1. **Convert 2 miles into feet.**

   Since 1 mile = 5,280 feet, the unit ratio is \( \frac{5,280 \text{ ft}}{1 \text{ mi}} \).

   
   
   \[
   2 \text{ mi} = 2 \text{ mi} \cdot \frac{5,280 \text{ ft}}{1 \text{ mi}} \quad \text{Multiply by } \frac{5,280 \text{ ft}}{1 \text{ mi}}. 
   \]

   
   \[
   = 2 \text{ mi} \cdot \frac{5,280 \text{ ft}}{1 \text{ mi}} \quad \text{Divide out common units.}
   \]

   
   \[
   = 2 \cdot 5,280 \text{ ft or 10,560 ft} \quad \text{Multiply.}
   \]

   So, 2 miles = 10,560 feet.

2. **ELEVATOR** The elevator in an office building has a weight limit posted of one and a half tons. How many pounds can the elevator safely hold?

   
   \[
   1 \frac{1}{2} \text{ t} = 1 \frac{1}{2} \text{ t} \cdot \frac{2,000 \text{ lb}}{1 \text{ t}} \quad \text{Multiply by } \frac{2,000 \text{ lb}}{1 \text{ t}}. 
   \]

   
   \[
   \quad \text{since there are 2,000 pounds in 1 ton.}
   \]

   
   \[
   = 1 \frac{1}{2} \cdot 2,000 \text{ lb or 3,000 lb} \quad \text{Multiply.}
   \]

   So, the elevator can safely hold 3,000 pounds.

**Check Your Progress**

**Complete.**

a. 8 yd = \( \square \) ft

   24

b. \( 4 \frac{1}{2} \text{ T} = \square \text{ lb} \)

   9,000
Convert Smaller Units to Larger Units

1 Convert 11 cups into pints.

Since 1 pint = 2 cups, the unit ratio is \( \frac{2\text{ c}}{1\text{ pt}} \), and its reciprocal is \( \frac{1\text{ pt}}{2\text{ c}} \).

\[
11\text{ c} = 11\text{ c} \cdot \frac{1\text{ pt}}{2\text{ c}} \quad \text{Multiply by } \frac{1\text{ pt}}{2\text{ c}}.
\]

\[
= 11 \cdot \frac{1\text{ pt}}{2}\quad \text{Divide out common units.}
\]

\[
= 11 \cdot \frac{1}{2}\\text{ pt}
\]

\[
= \frac{11}{2}\\text{ pt}
\]

Multiplying 11 by \( \frac{1}{2} \) is the same as dividing 11 by 2.

\[
= 5\frac{1}{2}\\text{ pt}
\]

So, 11 cups = \( 5\frac{1}{2} \) pints.

2 SOCCER Tracy kicked a soccer ball 1,000 inches. How many feet did she kick the ball?

Since 1 foot = 12 inches, multiply by \( \frac{1\text{ ft}}{12\text{ in.}} \). Then divide out common units.

\[
1,000\text{ in.} = 1,000\text{ in.} \cdot \frac{1\text{ ft}}{12}\text{ in.}
\]

\[
= 1,000\text{ in.} \cdot \frac{1}{12}\text{ ft}
\]

\[
= 83\frac{1}{3}\text{ ft}
\]

So, Tracy kicked the soccer ball \( 83\frac{1}{3} \) ft.
Check Your Progress Complete.

a. $21 \text{ qt} = \square \text{ gal}$  

$$\frac{5}{4}$$

b. $78 \text{ oz} = \square \text{ lb}$  

$$\frac{7}{8}$$

**EXAMPLE**

**LEMONADE** Paul made 6 pints of lemonade and poured it into 10 glasses equally. How many cups of lemonade did each glass contain?

Begin by converting 6 pints to cups.

$$6 \text{ pt} = 6 \text{ pt} \cdot \frac{2 \text{ c}}{1 \text{ pt}}$$

$$= 6 \cdot 2 \text{ cups} \text{ or } 12 \text{ cups}$$

Find the unit rate which gives the number of cups per glass.

$$\frac{12 \text{ cups}}{10 \text{ glasses}} = 6 \frac{1}{5} \text{ or } 1 \frac{1}{5} \text{ cups per glass}$$

**Check Your Progress** **CANDY** Tom has 3 pounds of candy he plans to divide evenly among himself and his 3 best friends. How many ounces of candy will each of them get?

12 oz

**HOMEWORK ASSIGNMENT**

Page(s):

Exercises:
**Main Idea**
- Change metric units of length, capacity, and mass.

**Build Your Vocabulary**
- The metric system is a **decimal** system of measures.
- The meter is the base unit of **length**.
- The liter is the base unit of **capacity**.
- The gram measures **mass**.
- The base unit of mass in the metric system is the **kilogram**.

**Examples**

**Convert Units in the Metric System**

1. **Complete 7.2 m = □ mm.**
   - To convert from meters to millimeters, **multiply** by 1,000.
   - $7.2 \times 1,000 = 7,200$
   - So, $7.2 \text{ m} = 7,200 \text{ mm}$.

2. **Complete 40 cm = □ m.**
   - To convert from centimeters to meters, **divide** by 100.
   - $40 \div 100 = 0.4$
   - So, $40 \text{ cm} = 0.4 \text{ m}$.

**Organize It**
- Under the metric units tab, take notes on how to convert metric units, include examples involving length, capacity, and mass.
**Check Your Progress** 

Complete. 

a. \(7.5 \text{ m} = \square \text{ cm}\)  

b. \(3400 \text{ mm} = \square \text{ m}\)  

\[
\begin{align*}
750 \\
3.4
\end{align*}
\]

**EXAMPLE**

**FARMS** A bucket holds 12.8 liters of water. Find the capacity of the bucket in milliliters.

You are converting from liters to milliliters. Since the bucket holds 12.8 liters, use the relationship \(1 \text{ L} = 1000 \text{ mL}\).

\[
1 \text{ L} = 1000 \text{ mL}
\]

Write the relationship.

\[
\begin{align*}
12.8 \times 1 \text{ L} &= 12.8 \times 1000 \text{ mL} \\
12.8 \text{ L} &= 12800 \text{ mL}
\end{align*}
\]

Multiply each side by 12.8 since you have 12.8 liters. 

To multiply 12.8 by 1,000, move the decimal point 3 places to the right.

So, the capacity of the bucket in milliliters is \(12800 \text{ mL}\).

**Check Your Progress** **BOOKS** A box of textbooks has a mass of 32,850 grams. What is the mass of the box in kilograms?

\(32.85 \text{ kg}\)
EXAMPLES  Convert Between Measurement Systems

Convert 7.13 miles to kilometers. Round to the nearest hundredth if necessary.

Use the relationship $1 \text{ mile} \approx 1.61 \text{ km}$.

$1 \text{ mi} \approx \frac{1.61}{1} \text{ km}$ Write the relationship.

$7.13 \times 1 \text{ mi} \approx 7.13 \times \frac{1.61}{1} \text{ km}$ Multiply each side by 7.13 since you have 7.13 mi.

$7.13 \text{ mi} \approx \frac{11.4793}{1} \text{ km}$ Simplify.

So, 7.13 miles is approximately 11.48 kilometers.

Convert 925.48 grams to pounds. Round to the nearest hundredth if necessary.

Since 1 pound $\approx \frac{453.6}{1} \text{ grams}$, the unit ratio is $\frac{\text{1 lb}}{453.6 \text{ g}}$.

$925.48 \text{ g} \approx \frac{925.48}{453.6} \text{ g} \cdot \frac{\text{1 lb}}{453.6 \text{ g}}$ Multiply by $\frac{1 \text{ lb}}{453.6 \text{ g}}$.

$\approx \frac{925.48}{453.6} \text{ lb}$ or $2.04 \text{ lb}$ Simplify.

So, 925.48 grams is approximately 2.04 pounds.

CHECK YOUR PROGRESS  Complete. Round to the nearest hundredth if necessary.

a. 8.15 gal = $\square$ L  

b. 5.75 m = $\square$ yd
Main Idea

Two quantities are proportional if they have a constant rate or ratio.

A proportion is an equation stating that two ratios or rates are equivalent.

In a proportion, a cross product is the product of the numerator of one ratio and the denominator of the other ratio.

Key Concept

Proportion: A proportion is an equation stating that two ratios are equivalent.

Example

Identify Proportional Relationships

**MATH** Before dinner, Mohammed solved 8 math problems in 12 minutes. After dinner, he solved 2 problems in 3 minutes. Is the number of problems he solved proportional to the time?

To identify proportional relationships, you can compare unit rates or compare ratios by comparing cross products. Let’s compare ratios by comparing cross products:

\[
\frac{\text{problems}}{\text{minutes}} = \frac{8}{12} = \frac{2}{3} = \frac{2}{3} \\
8 \cdot 3 = 12 \cdot 2 \\
24 = 24
\]

Since the cross products are equal, the number of problems solved is proportional to the time.
Check Your Progress  Determine if the quantities $30 for 12 gallons of gasoline and $10 for 4 gallons of gasoline are proportional.

Yes

EXAMPLES  Solve a Proportion

2 Solve \( \frac{5}{8} = \frac{18}{x} \).

\[
\frac{5}{8} = \frac{18}{x} \\
5 \cdot x = 8 \cdot 18 \\
x = \frac{144}{5} = 28.8
\]

3 Solve \( \frac{3.5}{14} = \frac{6}{n} \).

\[
\frac{3.5}{14} = \frac{6}{n} \\
3.5 \cdot n = 14 \cdot 6 \\
n = \frac{84}{3.5} = 24
\]

Check Your Progress  Solve each proportion.

a. \( \frac{9}{15} = \frac{k}{18} \)

b. \( \frac{4.6}{w} = \frac{4}{5} \)

10.8  5.75
**EXAMPLE** Draw a Diagram

**ROCK CLIMBING** A rock climber stops to rest at a ledge 90 feet above the ground. If this represents 75% of the total climb, how high above the ground is the top of the rock?

**UNDERSTAND** You know that 90 feet is 75% of the total height. You need to find the total height.

**PLAN** Draw a diagram showing the part already climbed.

**SOLVE**

![Diagram showing the part already climbed]

You know that $75\% ÷ 3 = 25\%$. If 75% of the total height is 90 feet, then 25% of the total height would be $90 ÷ 3$, or 30, feet. You know that $75\% + 25\% = 100\%$, so 90 feet + 30 feet = 120 feet, which is the height of the top of the rock.

**CHECK** Since 75%, or 0.75, of the total height is 90 feet, and $90 ÷ 120 = 0.75$, the solution checks.

**Check Your Progress** **INVENTORY** A retail store has taken inventory of 400 items. If this represents 80% of the total items in the store, what is the total number of items in the store?

500
Scale drawings and scale models are used to represent objects that are too large or too small to be drawn at actual size.

The scale gives the ratio that compares the measurements of the drawing to the real object.

**Example: Use a Map Scale**

**Maps** What is the actual distance between Portland and Olympia?

Step 1 Use a ruler to find the map distance between the two cities. The map distance is about 1.69 inches.

Step 2 Write and solve a proportion using the scale. Let $d$ represent the actual distance between the cities.

$$\frac{3}{8} \times d = 23 \times 1.69$$

Cross products.

$$0.375d = 3.887$$

Multiply. Write $\frac{3}{8}$ as a decimal.

$$d = \frac{103.7}{0.375}$$

Divide both sides by 0.375.

The distance between the cities is about 103.7 kilometers.
Check Your Progress  MAPS On a map of California, the distance between San Diego and Bakersfield is about $11\frac{2}{5}$ centimeters. What is the actual distance if the scale is 1 centimeter = 30 kilometers?

about 342 km

EXAMPLE Use a Blueprint Scale

ARCHITECTURE On the blueprint of a new house, each square has a side length of $\frac{1}{4}$ inch. If the length of a bedroom on the blueprint is $1\frac{1}{2}$ inches, what is the actual length of the room?

Write and solve a proportion.

\[
\begin{align*}
\text{Scale} & \quad \text{Length of Room} \\
\text{blueprint} & \quad \frac{1}{4} \text{ inch} \quad 1\frac{1}{2} \text{ inches} \\
\text{actual} & \quad 2\frac{1}{2} \text{ feet} \quad t \text{ feet}
\end{align*}
\]

\[
\frac{\frac{1}{4}}{2\frac{1}{2}} = \frac{1\frac{1}{2}}{t}
\]

Cross products

\[
\frac{1}{4} \cdot t = 2\frac{1}{2} \cdot 1\frac{1}{2}
\]

Multiply.

\[
\frac{1}{4} t = \frac{15}{4}
\]

Simplify.

\[
t = 15
\]

The length of the room is 15 feet.
Check Your Progress  On a blueprint of a new house, each square has a side length of \(\frac{1}{4}\) inch. If the width of the kitchen on the blueprint is 2 inches, what is the actual width of the room?

```
24 feet
```

**EXAMPLE**  Find a Scale Factor (parallels Example 4 in text)

3 Find the scale factor of a blueprint if the scale is \(\frac{1}{2}\) inch = 3 feet.

\[
\frac{\frac{1}{2}}{3\text{ feet}} = \frac{\frac{1}{2}}{36\text{ inches}}
\]

Convert 3 feet to \(\text{inches}\).

\[
= \frac{\frac{1}{2}}{\frac{36}{36}} \cdot \frac{1}{2} \text{ inch}
\]

Multiply by \(\frac{2}{2}\) to eliminate the fraction in the numerator.

\[
= \frac{1}{72}
\]

Divide out the common units.

The scale factor is \(\frac{1}{72}\). That is, each measure on the blueprint is \(\frac{1}{72}\) the actual measure.

**HOMEWORK ASSIGNMENT**

Page(s): 
Exercises:

\[
\frac{1}{48}
\]
**EXAMPLES**

### Percents as Fractions

#### NUTRITION
In a recent consumer poll, 41.8% of the people surveyed said they gained nutrition knowledge from family and friends. What fraction is this? Write in simplest form.

\[
41.8\% = \frac{41.8}{100} \quad \text{Write a fraction with a denominator of 100.}
\]

\[
= \frac{41.8}{100} \cdot \frac{10}{10} \quad \text{Multiply to eliminate the decimal in the numerator.}
\]

\[
= \frac{418}{1000} \quad \text{or} \quad \frac{209}{500} \quad \text{Simplify.}
\]

#### Write 12\(\frac{1}{2}\)% as a fraction in simplest form.

\[
12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} \quad \text{Write a fraction.}
\]

\[
= 12\frac{1}{2} \div 100 \quad \text{Divide.}
\]

\[
= \frac{25}{2} \div 100 \quad \text{Write 12\(\frac{1}{2}\)} as an improper fraction.
\]

\[
= \frac{25}{2} \times \frac{1}{100} \quad \text{Multiply by the reciprocal of 100.}
\]

\[
= \frac{25}{200} \quad \text{or} \quad \frac{1}{8} \quad \text{Simplify.}
\]

### Check Your Progress

**a. ELECTION** In a recent election, 64.8% of registered voters actually voted. What fraction is this? Write in simplest form.

\[
\frac{81}{125}
\]

**b. Write 62\(\frac{1}{2}\)% as a fraction in simplest form.**

\[
\frac{5}{8}
\]
EXAMPLES Fractions as Percents

PRODUCE In one shipment of fruit to a grocery store, 5 out of 8 bananas were still green. Find this amount as a percent.

\[
\frac{5}{8} = \frac{n}{100}
\]

Write a proportion.

\[
500 = 8n
\]

Find the cross products.

\[
\frac{500}{8} = \frac{8n}{8}
\]

Divide each side by 8.

\[
62\frac{1}{2} = n
\]

Simplify.

So, \(\frac{5}{8} = 62\frac{1}{2}\%\) or 62.5%.

Write \(\frac{5}{12}\) as a percent. Round to the nearest hundredth if necessary.

\[
\frac{5}{12} = \frac{n}{100}
\]

Write a proportion.

\[
500 = 12n
\]

Find the cross products.

\[
500 \div 12 \text{ ENTER } 41.66666667
\]

Use a calculator.

So, \(\frac{5}{12}\) is about 41.67%.

Write \(\frac{3}{7}\) as a percent. Round to the nearest hundredth.

\[
\frac{3}{7} = 0.4285714\ldots
\]

Write \(\frac{3}{7}\) as a decimal.

\[
= 42.86\% \quad \text{Multiply by 100 and add the } \%
\]

Check Your Progress Write each fraction as a percent. Round to the nearest hundredth.

a. \(\frac{13}{25}\)

52%

b. \(\frac{11}{15}\)

73.33%
6-1 Ratios

State whether each sentence is true or false. If false, replace the underlined word to make it a true sentence.

1. When you simplify a ratio, write a fraction as a mixed number.
   false; an improper fraction.

2. To write a ratio comparing measures, both quantities should have the same unit of measure.
   true

Write each ratio as a fraction in simplest form.

3. 63:7
   \[ \frac{9}{1} \]

4. 15:54
   \[ \frac{5}{18} \]

6-2 Rates

Complete.

5. A **rate** is a ratio that compares two quantities with different kinds of units.

Write each ratio as a fraction in simplest form.

6. 36 inches: 48 inches
   \[ \frac{3}{4} \]

7. 15 minutes to 3 hours
   \[ \frac{1}{12} \]
8. The table shows Amanda’s running time during a 5-mile race. Graph the data. Find the slope of the line. Explain what the slope represents.

<table>
<thead>
<tr>
<th>Distance (miles)</th>
<th>Time (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
</tr>
</tbody>
</table>

The slope is 6 and represents the time it took Amanda to run each mile.

9. \( \frac{3}{4} \) pt = ■ c

10. 90 ft = ■ yd

11. 156 oz = ■ lb

12. 4.3 cm = 43 mm

13. 42.7 g = 42,700 mg

14. The cross products of a proportion are equal.

15. If you know three parts of a proportion, you can solve for the fourth part by cross-multiplying and then dividing both sides by the coefficient of the unknown.

Solve each proportion.

16. \( \frac{15}{\phantom{1}} = \frac{3}{8} \) 40

17. \( \frac{6}{\phantom{0}} = \frac{8}{x} \) 24

18. \( \frac{b}{16} = \frac{3}{48} \) 1
Problem-Solving Investigation: Draw a Diagram

19. **LADDERS** A ladder leans against a wall. The top of the ladder rests against the wall at a point 12 feet above the ground. If this distance represents 80% of the height of the wall, how tall is the wall?

15 ft

Scale Drawings

On a map, the scale is $\frac{1}{4}$ inch = 10 miles. For each map distance, find the actual distance.

20. 6 inches 240 miles 21. $\frac{3}{8}$ inch 15 miles

22. $2\frac{1}{2}$ inches 100 miles 23. 1 inch 40 miles

Fractions, Decimals, and Percents

Complete the table of equivalent fractions.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>24. $\frac{1}{3}$</td>
<td>0.3</td>
<td>$33\frac{1}{3}$%</td>
</tr>
<tr>
<td>25. $\frac{3}{8}$</td>
<td>0.375</td>
<td>$37\frac{1}{2}$%</td>
</tr>
<tr>
<td>26. $\frac{1}{8}$</td>
<td>0.125</td>
<td>12.5%</td>
</tr>
<tr>
<td>27. $\frac{7}{8}$</td>
<td>0.875</td>
<td>$87\frac{1}{2}$%</td>
</tr>
</tbody>
</table>
ARE YOU READY FOR 
THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
  • You are probably ready for the Chapter Test.
  • You may want to take the Chapter 6 Practice Test on page 337 of your textbook as a final check.

☐ I used my Foldable or Study Notebook to complete the review of all or most lessons.
  • You should complete the Chapter 6 Study Guide and Review on pages 333–336 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 6 Practice Test on page 337 of your textbook.

☐ I asked for help from someone else to complete the review of all or most lessons.
  • You should review the examples and concepts in your Study Notebook and Chapter 6 Foldable.
  • Then complete the Chapter 6 Study Guide and Review on pages 333–336 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 6 Practice Test on page 337 of your textbook.

Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 6.

Student Signature

Parent/Guardian Signature

Teacher Signature

Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.
Applying Percents

Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

**Begin with a sheet of 11" x 17" paper.**

**STEP 1** Fold the paper in half lengthwise.

**STEP 2** Open and refold the paper into fourths along the opposite axis.

**STEP 3** Trace along the fold lines and label each section with a lesson title or number.

**NOTE-TAKING TIP:** When you take notes, it is often helpful to reflect on ways the concepts apply to your daily life.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 7. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>discount</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>percent equation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>percent of change</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>percent of decrease</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>percent of increase</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition</td>
<td>Description or Example</td>
</tr>
<tr>
<td>-----------------------</td>
<td>---------------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>percent proportion</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>principal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sales tax</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>simple interest</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Example 1
**Find the Percent of a Number**

#### Method 1
Write the percent as a fraction.

\[
8\% = \frac{8}{100} \text{ or } \frac{2}{25}
\]

\[
\frac{2}{25} \text{ of } 125 = \frac{2}{25} \times 125 = 10
\]

#### Method 2
Write the percent as a decimal.

\[
8\% = \frac{8}{100} \text{ or } 0.08
\]

\[
0.08 \text{ of } 125 = 0.08 \times 125 = 10
\]

So, 8\% of 125 is 10.

### Check Your Progress
Find 72\% of 350.

### Example 2
**Use Percents Greater than 100\%**

**Find 125\% of 64.**

You can either write the percent as a \text{fraction} or as a \text{decimal}.

\[
125\% = \frac{125}{100} = 1.25
\]

\[
1.25 \text{ of } 64 = 1.25 \times 64 = 80
\]

So, 125\% of 64 is 80.
**Check Your Progress**  Find 225% of 50.

112.5

**Example**

**LANGUAGES** The graph below shows that 30% of the people in a community speak Spanish as their first language. If a community has 800 people, how many people can be expected to speak Spanish as their first language?

To find 30% of 800, write the percent as a **decimal**. Then multiply.

\[
30\% \text{ of } 800 = 30\% \cdot 800 \\
= 0.30 \cdot 800 \\
= 240
\]

So, about **240** people in the community speak Spanish as their first language.

**Check Your Progress**  **SLEEP** The average person sleeps 33% of their adult life. If their adult life consists of 62 years, how many years does the average person spend sleeping?

20.46 years
**Main Idea**
- Solve problems using the percent proportion.

**Example**

**Find the Percent**

What percent of 24 is 18?

18 is the part, and 24 is the whole. You need to find the percent.

\[
\frac{p}{w} = \frac{n}{100}
\]

Write the proportion.

\[
\frac{18}{24} = \frac{n}{100}
\]

Find the cross products.

\[18 \cdot 100 = 24 \cdot n\]

Simplify.

\[1,800 = 24n\]

Divide each side by 24.

\[\frac{18}{24} = \frac{24n}{24}\]

Simplify.

\[75 = n\]

So, 75% of 24 is 18.

**Example**

**Find the Part**

What number is 30% of 150?

30 is the percent and 150 is the base. You need to find the part.

\[
\frac{p}{w} = \frac{n}{100}
\]

Percent proportion

\[
\frac{p}{150} = \frac{30}{100}
\]

Find the cross products.

\[p \cdot 100 = 150 \cdot 30\]

Simplify.

\[100p = 4,500\]

Divide each side by 100.

\[\frac{100p}{100} = \frac{4,500}{100}\]

Simplify.

\[p = 45\]

So, 30% of 150 is 45.
EXAMPLE

Find the Base

12 is 80% of what number?

12 is the part and 80 is the percent. You need to find the base.

\[
\frac{p}{w} = \frac{n}{100}
\]

Percent proportion

\[
\frac{12}{w} = \frac{80}{100}
\]

\[
a = 12, \quad n = 80.
\]

\[
12 \cdot 100 = w \cdot 80
\]

Find the cross products.

\[
1,200 = 80w
\]

Simplify.

\[
\frac{1,200}{80} = \frac{80w}{80}
\]

Divide each side by 80.

\[
15 = w
\]

So, 12 is 80% of 15.

Check Your Progress

a. What percent of 80 is 28?

35%

b. What number is 65% of 180?

117

c. 36 is 40% of what number?

90

WRITE IT

Write an example of a real-world percent problem.

______________

______________

______________

______________

HOMEWORK ASSIGNMENT

Page(s):

Exercises:

154  Math Connects, Course 2
Percent and Estimation

**EXAMPLE**

**CONCERTS** A town sold 407 tickets to a chamber music concert in the town square. Of the tickets sold, 61% were discounted for senior citizens. About how many senior citizens bought tickets for the concert?

You need to estimate 61% of 407.

61% is about 60%, and 407 is about 400.

$$61\% \text{ of } 407 \approx \frac{3}{5} \cdot 400 \quad 61\% \approx \frac{3}{5}$$

$$\approx 240$$ Multiply.

So, about 240 senior citizens bought tickets.

**Check Your Progress**

**TAXES** Michelle discovered that 27% of her paycheck was deducted for taxes. If her paycheck before taxes was $590, about how much was deducted for taxes?

about $$\frac{1}{4} \cdot 600$$ or $150

**EXAMPLE**

**COINS** Melinda calculated that 40% of the coins in her coin collection were minted before 1964. If there are 715 coins in her collection, about how many of them were minted before 1964?

You can use a fraction or 10% of a number to estimate. Let’s use 10% of a number.

**Step 1** Find 10% of the number.

715 is about 700.

$$10\% \text{ of } 700 = 0.1 \cdot 700 = 70$$

(continued on the next page)
Step 2  Multiply.

40% of 700 is $4 \times 10\%$ of 700.

$4 \times 70 = 280$

So, about $280$ coins were minted before 1964.

**Check Your Progress**  **SAVINGS**  Suki saves 70% of her monthly allowance. If her monthly allowance is $58, about how much does she save?

about $42$

**EXAMPLES**  **Percents Greater Than 100 or Less Than 1**

1. **Estimate 173% of 60.**

173% is about 175%.

$175\% \text{ of } 60 = (00\% \text{ of } 60) + (75\% \text{ of } 60)$

$= (1 \cdot 60) + (\frac{3}{4} \cdot 60)$

$= 60 + 45$ or $105$

So, 173% of 60 is about $105$.

2. **Estimate $\frac{1}{3}\%$ of 898.**

$\frac{1}{3}\%$ is one third of 1%. 898 is about 900.

$1\% \text{ of } 900 = 0.01 \cdot 900$

Write 1% as $0.01$.

$= 9$

Multiply.

One third of 9 is $\frac{1}{3} \cdot 9$ or $3$.

So, $\frac{1}{3}\%$ of 898 is about $3$.

**Check Your Progress**  **Estimate.**

a. 142% of 80  

about 112

b. $\frac{1}{5}\%$ of 197  

about 0.4
Main Idea
- Solve problems by using the percent equation.

Build Your Vocabulary (pages 149–150)
The equation \( \text{part} = \text{percent} \cdot \text{whole} \) is called the percent equation.

Example: Find the Part
1. What number is 46% of 200?

46% or \( 0.46 \) is the percent and \( 200 \) is the whole.

Let \( p \) represent the \( \text{part} \).

\[
\text{part} = \text{percent} \cdot \text{whole}
\]

\[
p = 0.46 \cdot 200 \quad \text{Write an equation.}
\]

\[
p = 92 \quad \text{Multiply.}
\]

So, 46% of 200 is 92.

Example: Find the Percent
2. 26 is what percent of 32?

Let \( n \) represent the percent.

\[
\text{part} = \text{percent} \cdot \text{whole}
\]

\[
\frac{92}{32} = \frac{n}{32} \quad \text{Write an equation.}
\]

\[
\frac{26}{32} = \frac{32n}{32} \quad \text{Divide each side by 32.}
\]

\[
0.8125 = n \quad \text{Simplify.}
\]

\[
81.25% = n \quad \text{Write as a percent.}
\]

So, 26 is \( 81.25\% \) of 32.
**EXAMPLE** Find the Whole

12 is 40% of what number?

Let \( w \) represent the whole.

\[
\frac{\text{part}}{\text{percent \cdot whole}} = \frac{12}{0.40 \cdot w}
\]

Write an equation.

\[
\frac{12}{0.40} = \frac{w}{0.40}
\]

Divide each side by 0.40.

\[
30 = w
\]

So, 12 is 40% of 30.

**Check Your Progress**

a. What number is 72% of 500?

360

b. 18 is what percent of 80?

22.5%

c. 36 is 90% of what number?

40

**WRITE IT**

Name two ways a percent can be written in the percent equation.

________________
________________
________________
________________
________________
**Problem-Solving Investigation:**

**Determine Reasonable Answers**

**MAIN IDEA**
- Solve problems by determining reasonable answers.

**EXAMPLE**

Solve. Use the Reasonable Answer Strategy.

**FUNDRAISER** A soccer team is having a candy sale to raise funds to buy new shirts. The team gets to keep 25% of the sales. Each candy bar costs $1.50, and the team has sold 510 bars so far. If the shirts cost a total of $175, should the team order the shirts yet? Explain.

**UNDERSTAND**
You know the shirts cost a total of $175 and that each candy bar costs $1.50. You know that the team has sold 510 bars so far and that they get to keep 25% of the sales. You need to know if the team has enough money to order the shirts yet.

**PLAN**
Find how much the team has earned so far.

Round 510 to 500. Then find 25% of their sales.

**SOLVE**

\[
1.50 \cdot 500 = 750
\]

Find 25% of $750.

\[
25\% \text{ of } 750 = 0.25 \cdot 750 = 187.50
\]

The team gets to keep $187.50. Since this is more than the cost of the shirts, they should order the shirts.

**CHECK**
Use a calculator to check that the actual result is 191.25, so the answer is reasonable.

**Check Your Progress**

**FIELD TRIP** There are 392 students in the seventh grade at Hamilton Middle School. If 35% of the seventh grade will attend the class field trip, is it reasonable to say that about 170 students will attend the field trip? Explain.

No, the answer is not reasonable. 35% of 392 ≈ 35% of 400 or 140.
A percent of change is a ratio that compares the change in quantity to the original amount.

If the original quantity is increased, the percent of change is called the percent of increase.

If the original quantity is decreased, the percent of change is called the percent of decrease.

**EXAMPLE** Find Percent of Increase

**SHOPPING** Last year a sweater sold for $56. This year the same sweater sells for $60. Find the percent of change in the cost of the sweater. Round to the nearest whole percent if necessary.

Since the new price is greater than the original price, this is a percent of increase. The amount of increase is

\[ 60 - 56 \text{ or } 4 \]

\[
\text{percent of increase} = \frac{\text{amount of increase}}{\text{original amount}}
\]

\[ = \frac{4}{56} \quad \text{Substitution} \]

\[ = 0.07 \quad \text{Simplify.} \]

\[ = 7\% \quad \text{Write as a percent.} \]

The percent of increase is about 7\%.
Check Your Progress **DVDs** Last year a DVD sold for $20. This year the same DVD sells for $24. Find the percent of change in the cost of the DVD. Round to the nearest whole percent if necessary.

20% increase

**EXAMPLE Find Percent of Decrease**

**ATTENDANCE** On the first day of school this year, 435 students reported to Howard Middle School. Last year on the first day, 460 students attended. Find the percent of change for the first day attendance. Round to the nearest whole percent if necessary.

Since the new enrollment figure is less than the figure for last year, this is a percent of decrease. The amount of decrease is $460 - 435$ or 25 students.

percent of decrease $= \frac{\text{amount of decrease}}{\text{original amount}}$

$= \frac{25}{460}$ Substitution

$= 0.05$ Simplify.

$= 5\%$ Write 0.05 as a percent.

The percent of decrease in the enrollment is about 5%.

Check Your Progress **ZOO** At the beginning of the summer season, the local zoo reported having 385 animals in its care. At the beginning of last year’s summer season the zoo had reported 400 animals. Find the percent of change in the number of animals at the zoo. Round to the nearest whole percent if necessary.

about 4% decrease
Sales Tax and Discount

**Main Idea**
- Solve problems involving sales tax and discount.

**Build Your Vocabulary** (pages 149–150)

- **Sales tax** is an **additional** amount of money charged on items that people **buy**.
- **Discount** is the amount by which the regular **price** of an item is **reduced**.

**Example**

**Find the Total Cost**

**GOLF**
A set of golf balls sells for $20, and the sales tax is 5.75%. What is the total cost?

To find the total cost, you can add sales tax to the regular price or add the percent of tax to 100%. Let’s add sales tax to the regular price.

First, find the **sales** tax.

\[
0.0575 \times 20 = 1.15
\]

The sales tax is **$1.15**.

Next, add the sales tax to the regular price.

\[
1.15 + 20 = 21.15
\]

The **total** cost of the set of golf balls is **$21.15**.

**Check Your Progress**

**BOOKS**
A set of three paperback books sells for $35 and the sales tax is 7%. What is the total cost of the set?

**$37.45**
EXAMPLE  Find the Sale Price

OUTERWEAR  Whitney wants to buy a new coat that has a regular price of $185. This weekend, the coat is on sale at a 33% discount. What is the sale price of the coat?

METHOD 1  First, find the amount of the discount $d$.

\[
33\% \text{ of } $185 = 0.33 \cdot 185 \quad \text{Write } 33\% \text{ as a decimal.}
\]

\[
= $61.05 \quad \text{The discount is } $61.05.
\]

So, the sale price is $185 - $61.05 or $123.95.

METHOD 2  First, subtract the percent of discount from 100%.

\[
100\% - 33\% = 67\%
\]

So, the sale price is 67\% of the regular price.

\[
67\% \text{ of } $185 = 0.67 \cdot 185 \quad \text{Write } 67\% \text{ as a decimal.}
\]

\[
= $123.95 \quad \text{Use a calculator.}
\]

So, the sale price of the coat is $123.95.

Check Your Progress  ELECTRONICS  Alex wants to buy a DVD player that has a regular price of $175. This weekend, the DVD player is on sale at a 20% discount. What is the sale price of the DVD player?

$140
EXAMPLE

Find the Percent of the Discount

Watches

A sports watch is on sale for $60.20 after a 30% discount. What is the original price?

First, find the percent paid.

100% - 30% = 70%

Next, use the percent equation to find the original price.

Words

$60.20 is 70% of what amount?

Variable

Let $n$ represent the original price.

Equation

$60.20 = 70\% \cdot n$

$60.20 = 0.70 \cdot n$ Write 70% as a decimal.

$86 = n$ Divide each side by 0.70.

The original price of the sports watch is $86.

Check Your Progress

Furniture

A rocking chair is on sale for $318.75 after a 15% discount. What is the original price?

$375
Simple Interest

**Main Idea**
- Solve problems involving simple interest.

**Build Your Vocabulary (pages 149–150)**

Simple Interest is the amount paid or earned for the use of money.

Principal is the amount of money deposited or invested.

**Examples**

**Find Interest Earned**

**Savings** Brandon found a bank offering a certificate of deposit that pays 4% simple interest. He has $1,500 to invest. How much interest will he earn in each amount of time?

1. **3 years**
   
   \[ I = prt \]
   
   Replace the variables.
   
   \[ I = 1,500 \cdot 0.04 \cdot 3 \]
   
   Simplify.
   
   \[ I = $180 \]
   
   Brandon will earn $180 in interest in 3 years.

2. **30 months**
   
   30 months \(= \frac{30}{12} = 2.5 \) years
   
   Write the time as years.
   
   \[ I = prt \]
   
   Replace the variables.
   
   \[ I = 1,500 \cdot 0.04 \cdot 2.5 \]
   
   Simplify.
   
   \[ I = 150 \]
   
   Brandon will earn $150 in interest in 30 months.
Check Your Progress

a. SAVINGS Cheryl opens a savings account that pays 5% simple interest. She deposits $600. How much interest will she earn in 2 years?

$60

b. SAVINGS Micah opens a savings account that pays 4% simple interest. He deposits $2,000. How much interest will he earn in 42 months?

$280

EXAMPLE

Find Interest Paid on a Loan

LOANS Laura borrowed $2,000 from her credit union to buy a computer. The interest rate is 9% per year. How much interest will she pay if it takes 8 months to repay the loan?

\[ I = \frac{prt}{12} \]

Replace \( p \) with $2,000, \( r \) with \( 0.09 \), and \( t \) with \( \frac{8}{12} \).

\[ I = 2,000 \cdot 0.09 \cdot \frac{8}{12} \]

Simplify.

Laura will pay $120 in interest in 8 months.

Check Your Progress

LOANS Juan borrowed $7,500 from the bank to purchase a used car. The interest rate is 15% per year. How much interest will he pay if it takes 2 years to repay the loan?

$2,250
BRINGING IT ALL TOGETHER

STUDY GUIDE

FOLDABLES

Use your Chapter 7 Foldable to help you study for your chapter test.

VOCABULARY PUZZLEMAKER

To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 7, go to:

BUILD YOUR VOCABULARY

You can use your completed Vocabulary Builder (pages 149–150) to help you solve the puzzle.

7-1

Percent of a Number

Find each number.

1. What is 3% of 530?
   
   159

2. Find 15% of $24.
   
   $3.60

3. Find 200% of 17.
   
   4.8

4. What is 0.6% of 800?
   
   34

7-2

The Percent Proportion

5. In the formula \( \frac{p}{w} = \frac{n}{100} \), \( p \) is the \( \text{part} \), \( w \) is the \( \text{whole} \), and \( n \) is the \( \text{percent} \).

6. What number is 30% of 15?
   
   4.5

7. 32.5 is 65% of what number?
   
   50
Percent and Estimation

Write fraction equivalents in simplest form for the following percents.

8. 20% \( \frac{1}{5} \)  
9. 40% \( \frac{2}{5} \)  
10. 60% \( \frac{3}{5} \)  
11. 80% \( \frac{4}{5} \)  
12. 25% \( \frac{1}{4} \)  
13. 50% \( \frac{1}{2} \)  
14. 75% \( \frac{3}{4} \)  
15. 100% \( 1 \)

Estimate.

16. 49% of 80 \( 40 \)  
17. 78% of 25 \( 20 \)  
18. 153% of 10 \( 15 \)  
19. 0.5% of 200 \( 1 \)

Algebra: The Percent Equation

Write an equation for each problem. Then solve.

20. 40% of what number is 48? \( 48 = 0.4n; 120 \)  
21. 18 is what percent of 72? \( 18 = n \cdot 72; 25\% \)  
22. Find 80% of 90. \( 0.8 \cdot 90 = n; 72 \)  
23. 12% of what number is 60? \( 0.12n = 60; 500 \)

Problem-Solving Investigation: Determine Reasonable Answers

24. TRAVEL The Winston family determined that lodging accounted for 48% of their total travel costs. If they spent $1,240 total during their trip, would about $560, $620, or $750 be a reasonable amount that they spent on lodging?

about $620
Percent of Change

State whether each sentence is true or false. If false, replace the underlined word to make a true sentence.

25. If the new amount is less than the original amount, then there is a percent of increase.
   false; decrease

26. The amount of increase is the new amount minus the original amount.
   true

Find the percent of change. Round to the nearest whole percent. State whether the percent of change is an increase or decrease.

27. original: $48; new: $44.25
   8%; percent of decrease

28. original: $157; new: $181
   15%; percent of increase

29. original: $17.48; new: $9.98
   43%; percent of decrease
Sales Tax and Discount

Find the total cost or sale price to the nearest cent.

30. $29.99 jeans; 15% discount  
   $25.49

31. $6.25 lunch; 8.5% sales tax  
   $6.78

Find the percent of discount to the nearest percent.

32. Pen: regular price, $9.95; sale price, $6.95  
   30%

33. Sweatshirt: regular price, $20; sale price, $15.95  
   20%

Simple Interest

Find the interest earned to the nearest cent for each principal, interest rate, and time.

34. $15,000, 9%, 2 years, 4 months  
   $3,150

35. $250, 3.5%, 6 years  
   $52.50
ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
  • You are probably ready for the Chapter Test.
  • You may want to take the Chapter 7 Practice Test on page 389 of your textbook as a final check.

☐ I used my Foldable or Study Notebook to complete the review of all or most lessons.
  • You should complete the Chapter 7 Study Guide and Review on pages 384–388 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 7 Practice Test on page 389 of your textbook.

☐ I asked for help from someone else to complete the review of all or most lessons.
  • You should review the examples and concepts in your Study Notebook and Chapter 7 Foldable.
  • Then complete the Chapter 7 Study Guide and Review on pages 384–388 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 7 Practice Test on page 389 of your textbook.

---

Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 7.
Statistics: Analyzing Data

Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with nine sheets of notebook paper.

**STEP 1** Fold 9 sheets of paper in half along the width.

**STEP 2** Cut a 1" tab along the left edge through one thickness.

**STEP 3** Glue the 1" tab down. Write the lesson number and title on the front tab.

**STEP 4** Repeat Steps 2 and 3 for the remaining sheets. Staple them together on the glued tabs to form a booklet.

**NOTE-TAKING TIP:** When you take notes, it is sometimes helpful to make a graph, diagram, picture, chart, or concept map that presents the information introduced in the lesson.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 8. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>analyze</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>bar graph</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>biased sample</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cluster</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>data</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>histogram</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>inferences</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>leaf</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>line graph</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>line plot</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(continued on the next page)
<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>measures of central tendency</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>median</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mode</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>outlier</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>population</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>random sample</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>range</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>scatter plot</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>statistics</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>stem</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>stem-and-leaf plot</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>survey</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>unbiased sample</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Statistics deals with collecting, organizing, and interpreting data.

A line plot is a diagram that shows the data on a number line.

Data that is grouped closely together is called a cluster.

Outliers are numbers that are quite separated from the rest of the data in a data set.

**EXAMPLE** Display Data Using a Line Plot

**PRESIDENTS** The table below shows the ages of the U.S. presidents at the time of their inaugurations. Make a line plot of the data.

<table>
<thead>
<tr>
<th>Age at Inauguration</th>
</tr>
</thead>
<tbody>
<tr>
<td>57 51 54 56 61 61 49 49 55 52 57 64 50 51 69</td>
</tr>
<tr>
<td>57 50 47 54 64 58 48 55 51 46 57 65 55 60 54</td>
</tr>
<tr>
<td>61 52 54 62 68 54 56 42 43 46 51 55 56</td>
</tr>
</tbody>
</table>

**Step 1** Draw a number line. Use a scale of 40 to 70 and an interval of 5.

**Step 2** Place an × above the number that represents the age of each U.S. president.

**Check Your Progress**

**STUDY TIME** The table at the right shows the number of minutes each student in a math class spent studying the night before the last math exam. Make a line plot of the data.

<table>
<thead>
<tr>
<th>Minutes Studying</th>
</tr>
</thead>
<tbody>
<tr>
<td>36 42 60 35</td>
</tr>
<tr>
<td>70 48 55 32</td>
</tr>
<tr>
<td>60 58 42 55</td>
</tr>
<tr>
<td>38 45 60 50</td>
</tr>
</tbody>
</table>

**Foldables**

**ORGANIZE IT** Write a set of data that could be displayed in a line plot. Under the lab for Lesson 8-1, display the data in a line plot.
**Build Your Vocabulary** (pages 173–174)

The range is the difference between the greatest and least numbers in the data set. When you analyze data, you use observations to describe and compare data.

**Example** Use a Plot to Analyze Data

**Climate** The line plot shows the number of inches of precipitation that fell in several cities west of the Mississippi River during a recent year. Identify any clusters, gaps, and outliers, and find the range of the data.

There are data clusters between 11 and 13 inches and between 16 and 18 inches. There are gaps: between 18 and 23; between 27 and 32.

Since 5 and 50 are apart from the rest of the data, they could be outliers.

The range is 50 – 5 or 45 inches.

**Check Your Progress** Age The line plot below shows the ages of students in an introductory computer course at the local community college. Identify any clusters, gaps, and outliers, and find the range of the data.

Clusters: between 18 and 27 years; Gap: between 30 and 44 years; Outlier: 44; Range: 26 years
Measures of central tendency can be used to describe the center of the data. The mean of a set of data is the sum of the data divided by the number of items in the data set.

**EXAMPLE** Find the Mean

**ANIMALS** The table below shows the number of species of animals found at 30 major zoos across the United States. Find the mean.

<table>
<thead>
<tr>
<th>Number of Species in Major U.S. Zoos</th>
</tr>
</thead>
<tbody>
<tr>
<td>300 400 283 400 175</td>
</tr>
<tr>
<td>617 700 700 715 280</td>
</tr>
<tr>
<td>800 290 350 133 400</td>
</tr>
<tr>
<td>195 347 488 435 640</td>
</tr>
<tr>
<td>232 350 300 300 400</td>
</tr>
<tr>
<td>705 400 800 300 659</td>
</tr>
</tbody>
</table>

Source: The World Almanac

\[
\text{mean} = \frac{300 + 400 + 283 + \ldots + 659}{30}
\]

The mean number of species of animals is 436.37

**Check Your Progress** SLEEP The table below shows the results of a survey of 15 middle school students concerning the number of hours of sleep they typically get each night. Find the mean.

<table>
<thead>
<tr>
<th>Nightly Hours of Sleep</th>
</tr>
</thead>
<tbody>
<tr>
<td>7  8  6  7  8</td>
</tr>
<tr>
<td>9  5  6  7  7</td>
</tr>
<tr>
<td>8  6  7  8  8</td>
</tr>
</tbody>
</table>

7.13 hours
**Build Your Vocabulary** (pages 173–174)

The **median** of a set of data is the middle number of the ordered data, or the mean of the middle two numbers.

The **mode** or modes of a set of data is the number or numbers that occur most often.

**Example** Find the Mean, Median, and Mode

**OLYMPICS** The table below shows the number of gold medals won by each country participating in the 2002 Winter Olympic games. Find the mean, median, and mode of the data.

<table>
<thead>
<tr>
<th>2002 Winter Olympics: Gold Medals Won</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 6 4 3 0</td>
</tr>
<tr>
<td>10 6 4 2 3</td>
</tr>
<tr>
<td>11 2 3 4 2</td>
</tr>
<tr>
<td>1 1 0 2 2</td>
</tr>
<tr>
<td>1 0 0 0 0</td>
</tr>
</tbody>
</table>

**mean**: sum of data divided by \(\frac{25}{25}\), or \(3.16\)

**median**: 13th number of the **ordered** data, or \(2\)

**mode**: number appearing **most** often, or \(0\)

**Check Your Progress** **PETS** The table below shows the number of pets students in an art class at Green Hills Middle School have at home. Find the mean, median, and mode of the data.

<table>
<thead>
<tr>
<th>Pets</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 2 1 0</td>
</tr>
<tr>
<td>1 3 5 2</td>
</tr>
<tr>
<td>0 1 0 2</td>
</tr>
<tr>
<td>3 1 2 0</td>
</tr>
</tbody>
</table>

**mean**: 1.44; **median**: 1; **mode**: 0
EXAMPLE

TEST EXAMPLE The average weight in pounds of several breeds of dogs is listed below.

15, 45, 26, 55, 15, 30

If the average weight of the Golden Retriever, 70 pounds, is added to this list, which of the following statements would be true?

A The mode would increase.
B The median would decrease.
C The median would increase.
D The mean would decrease.

Read the Item
You are asked to identify which statement would be true if the data value 70 was added to the data set.

Solve the Item
Use number sense to eliminate possibilities.

The mode, 15, will remain unchanged since the new data value occurs only once. So, eliminate choice A.

Since the new data value is greater than each value in the data set, neither the mean nor median will decrease. So, eliminate choices B and D.

Since 70 is greater than each value in the data set, the median will now increase. So, the answer is C.

Check Your Progress If the average weight of the Chihuahua, 4 pounds, is added to the list above, which of the following statements would be true?

F The mean would decrease.
G The mode would decrease.
H The median would stay the same.
J The mean would increase.

F
Build Your Vocabulary (pages 173–174)

Main Idea

- Display and analyze data in a stem-and-leaf plot.

In a stem-and-leaf plot, the data are organized from least to greatest. The digits of the least place value usually form the leaves and the next place-value digits form the stems.

Example Display Data in a Stem-and-Leaf Plot

Baseball The table below shows the number of home runs that Babe Ruth hit during his career from 1914 to 1935. Make a stem-and-leaf plot of the data.

<table>
<thead>
<tr>
<th>Home Runs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 54 25 46</td>
</tr>
<tr>
<td>4 59 47 41</td>
</tr>
<tr>
<td>3 35 60 34</td>
</tr>
<tr>
<td>2 41 54 6</td>
</tr>
<tr>
<td>11 22 46</td>
</tr>
<tr>
<td>29 46 49</td>
</tr>
</tbody>
</table>

Step 1 The digits in the least place value will form the leaves and the remaining digits will form the stems. In these data, 0 is the least value, and 60 is the greatest. So, the ones digit will form the leaves and the tens digit will form the stems.
Step 2  List the stems 0 to 6 in order from least to greatest in the *Stem* column. Write the leaves, the *ones* digits of the home runs, to the *right* of the corresponding stems.

Step 3  Order the leaves and write a *key* that explains how to read the stems and leaves.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 2 3 4 6</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2 5 9</td>
</tr>
<tr>
<td>3</td>
<td>4 5</td>
</tr>
<tr>
<td>4</td>
<td>1 1 6 6 6 7 9</td>
</tr>
<tr>
<td>5</td>
<td>4 4 9</td>
</tr>
<tr>
<td>6</td>
<td>0 2 5 = 25 home runs</td>
</tr>
</tbody>
</table>

A key shows how the digits are related.

The ones digits of the data form the leaves.

The tens digits of the data form the stems.

**Check Your Progress**  **BUSINESS** The table shows the number of hours several business men and women spent aboard an airplane. Make a stem-and-leaf plot of the data.

<table>
<thead>
<tr>
<th>Hours Aboard an Airplane</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 18 0231279</td>
</tr>
<tr>
<td>152690132210</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 0 4 6 6 7 9 9</td>
</tr>
<tr>
<td>1</td>
<td>0 1 2 3 4 5 8 9</td>
</tr>
<tr>
<td>2</td>
<td>1 2 3 6</td>
</tr>
<tr>
<td>3</td>
<td>5 13 = 13 hours</td>
</tr>
</tbody>
</table>
EXAMPLE Describe Data

**FITNESS** The stem-and-leaf plot below shows the number of miles that Megan biked each day during July. Find the range, median, and mode of the data.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5 5 5 6</td>
</tr>
<tr>
<td>1</td>
<td>0 0 0 0 1 2 2 5 8 8 9</td>
</tr>
<tr>
<td>2</td>
<td>1 2 5 8</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

25 miles

range: greatest distance — least distance = 30 — 5

or 25 miles

median: middle value, or 12 miles

mode: most frequent value, or 10 miles

**Check Your Progress** **SNOWFALL** The stem-and-leaf plot below shows the number of inches of snow that fell in Hightown during the month of January for the past 15 years. Find the range, median, and mode.

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1 3 5 7 9</td>
</tr>
<tr>
<td>1</td>
<td>0 0 0 2 4 4 7 8</td>
</tr>
<tr>
<td>2</td>
<td>2 6</td>
</tr>
</tbody>
</table>

2 inches

range: 25 inches; median: 10 inches; mode: 10 inches

WRITE IT

Evaluate how to find how many items are on a stem-and-leaf plot.

---

Math Connects, Course 2

Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.
EXAMPLE 3 Effects of Outliers

ANIMALS The average life span of several animal species is shown in the stem-and-leaf plot. Which measure of central tendency is most affected by the inclusion of the outlier?

The mode, 20, is not affected by the inclusion of the outlier, 40.

Calculate the mean and median each without the outlier, 40. Then calculate them including the outlier and compare.

without the outlier including the outlier

mean: \[ \frac{3 + 4 + \ldots + 22}{18} \approx 12.4 \]

\[ \frac{3 + 4 + \ldots + 20 + 40}{19} \approx 13.8 \]

median: 13.5 15

The mean increased by 13.8 - 12.4, or 1.4, while the median increased by 15 - 13.5, or 1.5. So, the median is most affected by the inclusion of the outlier.

Check Your Progress

TEST SCORES The test scores earned by a class of middle school math students on a chapter test are shown. Which measure of central tendency is most affected by the inclusion of the outlier?

Test Scores

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>5 6 7 9</td>
</tr>
<tr>
<td>8</td>
<td>0 0 1 2 2 2 5 5 6 6 7</td>
</tr>
<tr>
<td>9</td>
<td>0 2 3 3 3 4 4 6</td>
</tr>
</tbody>
</table>

7|5 = 75 points

mean
**Main Idea**

Display and analyze data using bar graphs and histograms.

---

**Build Your Vocabulary** (pages 173–174)

A bar graph is one method of comparing data by using solid bars to represent quantities.

---

**Example: Display Data Using a Bar Graph**

**Tourism** Make a bar graph to display the data in the table below.

<table>
<thead>
<tr>
<th>Country</th>
<th>Vacation Days per Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Italy</td>
<td>42</td>
</tr>
<tr>
<td>France</td>
<td>37</td>
</tr>
<tr>
<td>Germany</td>
<td>35</td>
</tr>
<tr>
<td>Brazil</td>
<td>34</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>28</td>
</tr>
<tr>
<td>Canada</td>
<td>26</td>
</tr>
<tr>
<td>Korea</td>
<td>25</td>
</tr>
<tr>
<td>Japan</td>
<td>25</td>
</tr>
<tr>
<td>United States</td>
<td>13</td>
</tr>
</tbody>
</table>

*Source: The World Almanac*

**Step 1** Draw and label the axes. Then choose a scale on the vertical axis so that it includes all of the vacation days per year.

**Step 2** Draw a bar to represent each category.
Check Your Progress

SPORTS  The table shows the average number of miles run each day during training by members of the cross country track team. Make a bar graph to display the data.

<table>
<thead>
<tr>
<th>Runner</th>
<th>Miles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bob</td>
<td>9</td>
</tr>
<tr>
<td>Tamika</td>
<td>12</td>
</tr>
<tr>
<td>David</td>
<td>14</td>
</tr>
<tr>
<td>Anne</td>
<td>8</td>
</tr>
<tr>
<td>Jonas</td>
<td>5</td>
</tr>
<tr>
<td>Hana</td>
<td>10</td>
</tr>
</tbody>
</table>

Miles Run Daily

Build Your Vocabulary (pages 173–174)

A histogram is a special kind of bar graph that uses bars to represent the frequency of numerical data that have been organized in intervals.

Write It

Explain when you would use a bar graph and when you would use a histogram.

EXAMPLE  Display Data Using a Histogram

BASKETBALL  The number of wins for 29 teams of a basketball league for a season have been organized into a frequency table. Make a histogram of the data.

<table>
<thead>
<tr>
<th>Number of Wins</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>11–20</td>
<td>3</td>
</tr>
<tr>
<td>21–30</td>
<td>4</td>
</tr>
<tr>
<td>31–40</td>
<td>4</td>
</tr>
<tr>
<td>41–50</td>
<td>10</td>
</tr>
<tr>
<td>51–60</td>
<td>8</td>
</tr>
</tbody>
</table>
Step 1 Draw and label horizontal and vertical axes.

Add a title.

Step 2 Draw a bar to represent the frequency of each interval.

Check Your Progress SPEED The speeds of cars on a stretch of interstate are clocked by a police officer and have been organized into a frequency table. Make a histogram of the data.

<table>
<thead>
<tr>
<th>Speed (mph)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>50–59</td>
<td>2</td>
</tr>
<tr>
<td>60–69</td>
<td>14</td>
</tr>
<tr>
<td>70–79</td>
<td>18</td>
</tr>
<tr>
<td>80–89</td>
<td>3</td>
</tr>
</tbody>
</table>
## Analyze Data to Make Inferences

### DINING OUT

The bar graph shows the number of times people dine out each month.

![Bar graph showing dining out data]

#### 3. How many people are represented in the histogram? Justify your answer.

Find the sum of the heights of the bars in the histogram.

\[
5 + 6 + 12 + 15 + 7 + 5 = 50
\]

#### 4. What percent of people surveyed ate out more than 40 times per month?

\[
\frac{7 + 5}{50} = \frac{12}{50}
\]

number of people who ate out more than 40 times

\[
\frac{12}{50} = 0.24
\]

Write the fraction as a decimal.

\[
0.24 = 24\%
\]

Write the decimal as a percent.

So, 24% of the people surveyed ate out more than 40 times per month.

### Check Your Progress

#### HOUSING

The bar graph shows the number of houses sold in various price ranges.

![Bar graph showing housing prices]

#### a. How many houses are represented in the histogram?

80

#### b. What percent of houses were sold for more than $200,000?

About 56%
EXAMPLE Solve Problems by Using a Graph

**VCR SALES** Based on the information in the graph, how many VCRs would you expect to be sold in 2012?

**UNDERSTAND** You know that the graph shows a rapid downward trend. You need to determine how many VCRs would be expected to be sold in 2012.

**PLAN** Look at the trend of the graph. Predict the number of VCR sales in 2012.

**SOLVE** If the trend continues, no VCRs will be expected to be sold in 2012.

**CHECK** The graph rapidly decreases. The answer is reasonable.

The graph shows a rapid downward trend. If it continued, no VCRs would be sold in 2012.

**Check Your Progress** **TEMPERATURE** Refer to the graph below. Suppose the trends continue. Predict the average high temperature for the month of August.

The graph shows an upward trend. If it continues, the average high temperature in August will be around 95 degrees.
Main Idea
- Analyze line graphs and scatter plots to make predictions and conclusions.

Build Your Vocabulary (pages 173–174)

Line graphs can be useful in predicting future events when they show trends over time or relationships.

Example: Use a Line Graph to Predict

Typing

The line graph shows the time it has taken Enrique to type a class paper so far. The paper is 600 words long. Use the graph to predict the total time it will take him to type his paper.

By looking at the pattern in the graph, you can predict that it will take Enrique about 14 minutes to type his 600-word paper.

Check Your Progress

Travel

During a recent road trip, Helen kept track of the number of miles traveled after each hour of travel time was completed. The table shows her information. Use the line graph to predict how far Helen will travel in 12 hours of travel time.

In 12 hours, Helen will travel about 700 miles.
A scatter plot displays two sets of data on the same graph and are also useful in making predictions.

**EXAMPLE** Use a Scatter Plot to Predict

**POLLUTION** The scatter plot shows the number of days that a city failed to meet air quality standards from 2000 to 2008. Use it to predict the number of days of bad air quality in 2014.

By looking at the pattern, you can predict that the number of days of bad air quality in 2014 will be about 48 days.

**Check Your Progress** GAS MILEAGE Use the scatter plot below to predict the gas mileage for a car weighing 5500 pounds.

By looking at the pattern, you can predict that the number of days of bad air quality in 2014 will be about 48 days.

**GAS MILEAGE** Use the scatter plot below to predict the gas mileage for a car weighing 5500 pounds.

20 mpg
8–7 Using Data to Predict

**Main Idea**
- Predict actions of a larger group by using a sample.

**Build Your Vocabulary** (pages 173–174)

A survey is designed to collect data about a specific group of people, called the population.

**Example**

PETS The table shows the results of a survey in which people were asked whether their house pets watch television. There are 540 students at McCloskey Middle School who own pets. Predict how many of them would say their pets watch TV.

<table>
<thead>
<tr>
<th>Response</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>yes</td>
<td>38%</td>
</tr>
<tr>
<td>no</td>
<td>60%</td>
</tr>
<tr>
<td>don’t know</td>
<td>2%</td>
</tr>
</tbody>
</table>

You can use the percent proportion and the survey results to predict the number of people who said their pets watch TV.

\[
\frac{p}{w} = \frac{n}{100}
\]

\[
\frac{p}{540} = \frac{38}{100}
\]

Cross products

\[
100a = 540(38)
\]

Simplify

\[
a = 205.2
\]

About 205 of the people surveyed said that their pets watch television.
SUMMER JOBS

According to one survey, 25% of high school students reported they would not get summer jobs. Predict how many of the 948 students at Mohawk High School will not get summer jobs.

You need to predict how many of the 948 students will not get summer jobs.

Words
What number is 25% of 948?

Variable
Let $n$ represent the number.

Equation
\[ n = 0.25 \times 948 \]

\[ n = 237 \]

Multiply.

So, you could predict that about 237 of the students at Mohawk High School will not get summer jobs.

Check Your Progress

SEASONS

According to one survey, 31% of adults consider spring to be their favorite season of the year. Predict how many of the 525 employees of a large corporation would respond that spring is their favorite season of the year.

about 163
Main Idea

- Predict the actions of a larger group by using a sample.

Build Your Vocabulary (pages 173–174)

A sample is representative of a larger population. An unbiased sample is representative of the entire population. A simple random sample is the most common type of unbiased sample.

A biased sample occurs when one or more parts of the population are favored over others. A convenience sample includes members of a population who are easily accessed. A voluntary response sample involves only those who want to participate in sampling.

Example: Determine Validity of Conclusions

Determine whether the conclusion is valid. Justify your answer.

A newspaper asks its readers to answer a poll about whether or not an issue should be on the ballot in an upcoming election. 85% of the readers who responded said that they wanted the issue on the ballot, so the newspaper printed an article saying that 85% of people want the issue on the ballot.

The conclusion is not valid. The population is restricted to readers and it is a voluntary response sample and is biased. The results of a voluntary response sample do not necessarily represent the entire population.
Check Your Progress Determine whether the conclusion is valid. Justify your answer.

A coffee shop asks every tenth customer that comes in the door to identify their favorite coffee drink. 45% of the customers surveyed said the mocha coffee is their favorite drink. The manager of the store concluded that about half of the store’s customers like the mocha coffee.

The conclusion is valid. The sample is an unbiased random sample.

Example

VENDING MACHINES An office building manager interviewed 60 of their employees to determine whether or not a vending machine should be placed in the break room. 45 of the employees said yes and 15 said no. If there are 255 employees in the building, predict how many employees would like a vending machine in the break room. (parallels Example 3 in text)

The sample is an unbiased random sample since employees were randomly selected. Thus, the sample is valid.

\[ \frac{45}{60} = \frac{75}{100} \] or \( 75\% \) of the employees would like a vending machine in the break room. So, find 75\% of \( \frac{255}{1} \).

\[ 0.75 \times 255 = 191.25 \]

75\% of 255 = 0.75 \( \times \) 255

So, about 191 employees would like a vending machine in the break room.

Check Your Progress CLUBS A Spanish teacher is trying to determine if students would be interested in joining a Spanish club. She randomly asked 30 of her students. 18 of the students said yes and 12 said no. If the teacher has 105 students in her Spanish classes, predict how many would like to join a Spanish club.

\[ 63 \]
**Main Idea**
- Recognize when statistics and graphs are misleading.

### Example
#### Changing the Interval of Graphs

**Business** The line graphs below show the last 10 weeks of sales for the Crumby Cookie Bakery.

![Graph A](image1)

![Graph B](image2)

**a.** Do the graphs show the same data? If so, explain how the graphs differ.

The graphs show the **same** data. However, the graphs differ in that Graph A has greater intervals and a greater range.

**b.** Which graph makes it appear that the bakery’s sales declined only slightly?

Graph A makes it appear that the sales declined only slightly even though both graphs show the same decline.

### Check Your Progress

**Soccer** The graphs show the number of wins by four different soccer teams. Do the graphs show the same data? If so, explain how they differ.

![Graph A](image3)

![Graph B](image4)

The graphs show the same data. The graphs differ in that the interval for Graph A is 1 unit and the interval for Graph B is 2 units. Also, the scale of Graph A starts at 6.
**EXAMPLE** Misleading Statistics  (parallels Example 3 in text)

**GRADES** Michael and Melissa both claim to be earning a C average, 70% to 79%, in their Latin class. One student is wrong. Which one? Explain how he or she is using a misleading statistic.

- **mean**
  - Michael: 53.4%
  - Melissa: 71.9%
- **median**
  - Michael: 70%
  - Melissa: 70%

<table>
<thead>
<tr>
<th>Test</th>
<th>Grade (%)</th>
<th>Michael</th>
<th>Melissa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80</td>
<td>88</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>76</td>
<td>83</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>73</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>70</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>40</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>25</td>
<td>65</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>62</td>
<td></td>
</tr>
</tbody>
</table>

Michael is wrong. He is using the **median** to describe his grade rather than the **mean**. Only Melissa’s mean, or average, is 70% or better.

**Check Your Progress** RETAIL SALES Two different grocery stores each claim to have the lowest average prices. Use the table to explain their reasoning and determine which store really has the lowest average prices.

<table>
<thead>
<tr>
<th>Item</th>
<th>Store A</th>
<th>Store B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milk</td>
<td>$1.29</td>
<td>$1.34</td>
</tr>
<tr>
<td>Bread</td>
<td>$1.99</td>
<td>$1.85</td>
</tr>
<tr>
<td>Eggs</td>
<td>$1.19</td>
<td>$1.09</td>
</tr>
<tr>
<td>Soda</td>
<td>$2.29</td>
<td>$2.99</td>
</tr>
<tr>
<td>Coffee</td>
<td>$7.99</td>
<td>$5.29</td>
</tr>
<tr>
<td>Ice Cream</td>
<td>$4.39</td>
<td>$4.19</td>
</tr>
</tbody>
</table>

Store A: mean = $3.19, median = $2.14; Store B: mean = $2.79, mean = $2.42. Store A is using the median to describe its average prices rather than the mean. Store B has the lowest average price.
8-1

Line Plots

The line plot shows prices for different running shoes.

1. What is the range of the prices? $90

8-2

Measures of Central Tendency and Range

Find the mean, median, and mode of each set of data.

2. 2, 5, 5, 6, 8, 11, 12
   mean: 7; median: 6; mode: 5

3. 6, 5, 12, 34, 20, 17
   mean: 15.7; median: 14.5; mode: none

8-3

Stem-and-Leaf Plots

4. The stem-and-leaf plot shows test scores for 13 students. Find the range, median, and mode of the data.
   range: 17; median: 20; mode: 23
8-4

Bar Graphs and Histograms

Write true or false for each statement. If the statement is false, replace the underlined words with words that will make the statement true.

5. A bar graph is used to compare data.
   true

6. A histogram shows categories on one of the axes.
   false; frequency intervals

8-5

Problem-Solving Investigation: Use a Graph

The graph shows the results of a survey about favorite countries students would like to visit.

7. Which place was favored by most students? Mexico

8. Compare the number of students that would like to visit Italy versus Ireland.
   Half as many students would like to visit Ireland.

8-6

Using Graphs To Predict

Refer to the graph shown.

9. Mark the City Zoo graph to show how to predict the attendance in 2005.

10. If the trend continues, predict the attendance in 2005. 11,000
Using Data To Predict

11. **LUNCHES** A survey of 7th graders showed that 44% bring their lunch to school. Predict how many of the 450 7th graders bring their lunch to school.

12. **ZOO** A survey of zoo visitors showed that 28% chose the lion exhibit as their favorite. If 338 people visited today, predict how many would choose the lion exhibit as their favorite.

Using Sampling To Predict

Determine whether each conclusion is valid. Justify your answer.

13. A researcher randomly surveys ten employees from each department of a large company to determine the number of employees that buy their lunch in the cafeteria. Of these, 82% said they do buy their lunch in the cafeteria. The researcher concludes that most of the employees do buy their lunch in the cafeteria.

The conclusion is valid. This is an unbiased random sample.

14. Every tenth customer who purchases books from an online store is asked to take a survey. The majority of those who replied said they would like more shipping options. As a result, the store adds more shipping options for their customers.

The conclusion is invalid. This is a biased, voluntary response survey.

Misleading Statistics

The table lists the number of wrong answers a student had on her homework papers this year.

15. Which measure of central tendency might she use to emphasize her good work? **mode**


The mean, 5, or the median, 6, better represent the data because they are close to the majority of the data values. The mode is not close to the other data values.
Are You Ready for the Chapter Test?

Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
  • You are probably ready for the Chapter Test.
  • You may want to take the Chapter 8 Practice Test on page 455 of your textbook as a final check.

☐ I used my Foldables or Study Notebook to complete the review of all or most lessons.
  • You should complete the Chapter 8 Study Guide and Review on pages 450–454 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may want to take the Chapter 8 Practice Test on page 455 of your textbook.

☐ I asked for help from someone else to complete the review of all or most lessons.
  • You should review the examples and concepts in your Study Notebook and Chapter 8 Foldables.
  • Then complete the Chapter 8 Study Guide and Review on pages 450–454 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 8 Practice Test on page 455 of your textbook.

Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 8.

Student Signature

Parent/Guardian Signature

Teacher Signature
Probability

Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

**Begin with five sheets of 8\(\frac{1}{2}\)" by 11" paper.**

**STEP 1** Stack 5 sheets of paper \(\frac{3}{4}\) inch apart.

**STEP 2** Roll up bottom edges so that all tabs are the same size.

**STEP 3** Crease and staple along fold.

**STEP 4** Write the chapter title on the front. Label each tab with a lesson number and title. Label the last tab *Vocabulary*.

**NOTE-TAKING TIP:** When taking notes, writing a paragraph that describes the concepts, the computational skills and the graphics will help you to understand the math in a lesson.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 9. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description of these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>combination</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>complementary events</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[KAHM-pluh-MEHN-tuh-ree]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>composite events</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>experimental probability</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[ihk-SPEHR-uh-MEHN-tuhl]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fair game</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fundamental Counting Principle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition</td>
<td>Description or Example</td>
</tr>
<tr>
<td>---------------------------------</td>
<td>---------------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>independent event</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>outcome</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>permutation [PUHR-myu-TAY-shuhn]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>probability [PRAH-buh-BIH-luh-tee]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>random</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sample space</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>simple event</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>theoretical probability [thee-uh-REHT-uh-kuhl]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tree diagram</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
An outcome is any possible result.

A simple event is one outcome or a collection of outcomes. Outcomes occur at random if each outcome occurs by chance.

**EXAMPLE** Find Probability

If the spinner shown is spun once, what is the probability of its landing on an odd number?

\[
P(\text{odd number}) = \frac{\text{odd numbers possible}}{\text{total numbers possible}}
\]

Two numbers are odd: 1 and 3.

\[
= \frac{2}{4} = \frac{1}{2}
\]

Simplify.

The probability of spinning an odd number is \(\frac{1}{2}\) or 50%.

**Check Your Progress** What is the probability of rolling a number less than three on a number cube marked with 1, 2, 3, 4, 5, and 6 on its faces?

\[\frac{1}{3}\]
EXAMPLE

GAMES A game requires spinning the spinner shown in Example 1. If the number spun is greater than 3, the player wins. What is the probability of winning the game?

Let \( P(A) \) be the probability that the player will win.

\[
P(A) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} = \frac{1}{4}
\]

The probability of winning the game is \( \frac{1}{4} \).

BUILD YOUR VOCABULARY (pages 202–203)

The sum of the probabilities of complementary events is 1 or 100%.

EXAMPLE

GAMES What is the probability of not winning the game described in Example 2?

\[
P(A) + P(\text{not } A) = 1 \quad \text{Definition of complementary events}
\]

\[
\frac{1}{4} + P(\text{not } A) = 1 \quad \text{Replace } P(A) \text{ with } \frac{1}{4}.
\]

\[
- \frac{1}{4} \quad - \frac{1}{4} \quad \text{Subtract } \frac{1}{4} \text{ from each side.}
\]

\[
P(\text{not } A) = \frac{3}{4}
\]

The probability of not winning the game is \( \frac{3}{4} \).

Check Your Progress A game requires spinning the spinner shown in Example 1. If the number spun is less than or equal to 2, the player wins.

a. What is the probability of winning the game?

b. What is the probability of not winning the game?
The sample space is the set of all possible outcomes.

A tree diagram can be used to display the sample space.

**EXAMPLE** Find the Sample Space

**CHILDREN** A couple would like to have two children. Find the sample space of the children’s genders if having a boy is equally likely as having a girl.

Make a table that shows all of the possible outcomes.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>girl</td>
<td>girl</td>
</tr>
<tr>
<td>girl</td>
<td>boy</td>
</tr>
<tr>
<td>boy</td>
<td>boy</td>
</tr>
<tr>
<td>boy</td>
<td>girl</td>
</tr>
</tbody>
</table>

**Check Your Progress** CARS A dealer sells a car in red, black, or white. The car also can be 2-door or 4-door. Find the sample space for all possible cars available from this dealer.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>red</td>
<td>2-door</td>
</tr>
<tr>
<td>black</td>
<td>2-door</td>
</tr>
<tr>
<td>white</td>
<td>2-door</td>
</tr>
<tr>
<td>red</td>
<td>4-door</td>
</tr>
<tr>
<td>black</td>
<td>4-door</td>
</tr>
<tr>
<td>white</td>
<td>4-door</td>
</tr>
</tbody>
</table>
TEST EXAMPLE  Amy was trying to decide what kind of sandwich to make. She had two kinds of bread, wheat and sourdough. And she had three kinds of lunchmeat, ham, turkey, and roast beef. Which list shows all the possible bread-lunchmeat combinations?

A

<table>
<thead>
<tr>
<th>Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>wheat ham</td>
</tr>
<tr>
<td>sourdough turkey</td>
</tr>
<tr>
<td>wheat turkey</td>
</tr>
<tr>
<td>sourdough ham</td>
</tr>
</tbody>
</table>

B

<table>
<thead>
<tr>
<th>Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>wheat ham</td>
</tr>
<tr>
<td>wheat turkey</td>
</tr>
<tr>
<td>wheat roast beef</td>
</tr>
</tbody>
</table>

C

<table>
<thead>
<tr>
<th>Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>wheat ham</td>
</tr>
<tr>
<td>wheat turkey</td>
</tr>
<tr>
<td>wheat roast beef</td>
</tr>
<tr>
<td>sourdough ham</td>
</tr>
<tr>
<td>sourdough turkey</td>
</tr>
<tr>
<td>sourdough roast beef</td>
</tr>
</tbody>
</table>

D

<table>
<thead>
<tr>
<th>Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>wheat turkey</td>
</tr>
<tr>
<td>sourdough turkey</td>
</tr>
<tr>
<td>wheat turkey</td>
</tr>
<tr>
<td>sourdough ham</td>
</tr>
<tr>
<td>wheat ham</td>
</tr>
<tr>
<td>sourdough ham</td>
</tr>
</tbody>
</table>

Read the Item

There are two bread choices and three lunchmeat choices. Find all of the bread-lunchmeat combinations.

(continued on the next page)
**Solve the Item**

Make a tree diagram to show the sample space.

**Tree Diagram**

- **Wheat**
  - Ham
  - Turkey
  - Roast Beef

- **Sourdough**
  - Turkey
  - Roast Beef

There are 6 different bread-lunchmeat combinations.

The answer is **C**.

**Check Your Progress**  
**MULTIPLE CHOICE** A new car can be ordered with exterior color choices of black, red, and white, and interior color choices of tan, gray, and blue. Which list shows the different cars that are possible?

- **F**
  - black, tan
  - red, tan
  - white, tan
  - black, gray
  - red, gray
  - white, gray
  - black, blue
  - red, blue
  - white, blue

- **H**
  - black, tan
  - red, gray
  - white, blue
  - black, gray
  - red, blue
  - white, tan

- **G**
  - black, tan
  - red, gray
  - white, blue
  - black, gray

- **J**
  - black, tan
  - red, gray
  - white, blue

**HOMEWORK ASSIGNMENT**

Page(s):

Exercises:

**WRITE IT**

In a probability game using two counters A and B, what would the outcome BA mean?
You can use the Fundamental Counting Principle to find the number of possible outcomes in a sample space.

**EXAMPLE** (parallels Example 2 in text)

**CLOTHING** The table below shows the shirts, shorts, and shoes in Gerry's wardrobe. How many possible outfits—one shirt, one pair of shorts, and one pair of shoes—can he choose?

<table>
<thead>
<tr>
<th>Shirts</th>
<th>Shorts</th>
<th>Shoes</th>
</tr>
</thead>
<tbody>
<tr>
<td>red</td>
<td>beige</td>
<td>black</td>
</tr>
<tr>
<td>blue</td>
<td>green</td>
<td>brown</td>
</tr>
<tr>
<td>white</td>
<td>blue</td>
<td></td>
</tr>
<tr>
<td>yellow</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \text{shirts} \times \text{shorts} \times \text{shoes} = \text{total} \]

\[4 \times 3 \times 2 = 24\]

There are 24 possible outfits that Gerry can choose.

**Check Your Progress** **SANDWICHES** The table below shows the types of bread, types of cheese, and types of meat that are available to make a sandwich. How many possible sandwiches can be made by selecting one type of bread, one type of cheese, and one type of meat?

<table>
<thead>
<tr>
<th>Bread</th>
<th>Cheese</th>
<th>Meat</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>American</td>
<td>Turkey</td>
</tr>
<tr>
<td>Wheat</td>
<td>Swiss</td>
<td>Ham</td>
</tr>
<tr>
<td>Rye</td>
<td>Mozzarella</td>
<td>Roast Beef</td>
</tr>
</tbody>
</table>
**Main Idea**
- Find the number of permutations of a set of objects and find probabilities.

**Build Your Vocabulary** (pages 202–203)

A permutation is an arrangement or listing of objects in which order is important.

**Example** Find a Permutation

**Bowling** A team of bowlers has five members, who bowl one at a time. In how many orders can they bowl?

There are 5 choices for the first bowler.

There are 4 choices for the second bowler.

There are 3 choices for the third bowler.

There are 2 choices for the fourth bowler.

There is 1 choice that remains.

\[ 5 \times 4 \times 3 \times 2 \times 1 = 120 \]

There are 120 possible arrangements, or permutations, of the five bowlers.

**Check Your Progress** Track and Field A relay team has four members who run one at a time. In how many orders can they run?

24
EXAMPLE

Find a Permutation

RAFFLE A school fair holds a raffle with 1st, 2nd, and 3rd prizes. Seven people enter the raffle, including Marcos, Lilly, and Heather. What is the probability that Marcos will win the 1st prize, Lilly will win the 2nd prize, and Heather will win the 3rd prize?

There are 7 choices for 1st prize.
There are 6 choices for 2nd prize.
There are 5 choices for 3rd prize.

\[7 \cdot 6 \cdot 5 = 210\] The number of permutations of 3 prizes.

There are 210 possible arrangements, or permutations, of the 3 prizes. Since there is only one way of arranging Marcos first, Lilly second, and Heather third, the probability of this event is \(\frac{1}{210}\).

Check Your Progress

CLUBS The president and vice president of the French Club will be randomly selected from a jar of 24 names. Find the probability that Sophie will be selected as president and Peter selected as vice president.

\(\frac{1}{552}\)
**Main Idea**
- Find the number of combinations of a set of objects and find probabilities.

**Build Your Vocabulary** (pages 202–203)

An arrangement, or listing, of objects in which order is not important is called a **combination**.

**Example**

**Find the Number of Combinations**

**Decorating** Ada can select from seven paint colors for her room. She wants to choose two colors to paint stripes on her walls. How many different pairs of colors can she choose?

**Method 1** Make a list.

Number the colors 1 through 7.

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2</td>
<td>1, 5</td>
<td>2, 3</td>
<td>2, 6</td>
<td>3, 5</td>
<td>4, 5</td>
<td>5, 6</td>
</tr>
<tr>
<td>1, 3</td>
<td>1, 6</td>
<td>2, 4</td>
<td>2, 7</td>
<td>3, 6</td>
<td>4, 6</td>
<td>5, 7</td>
</tr>
<tr>
<td>1, 4</td>
<td>1, 7</td>
<td>2, 5</td>
<td>3, 4</td>
<td>3, 7</td>
<td>4, 7</td>
<td>6, 7</td>
</tr>
</tbody>
</table>

There are 21 different pairs of colors.

**Method 2** Use a permutation.

There are $7 \cdot 6$ permutations of two colors chosen from seven. There are $2 \cdot 1$ ways to arrange the two colors.

$$\frac{7 \cdot 6}{2 \cdot 1} = \frac{42}{2} = 21$$

There are 21 different pairs of colors Ada can choose.

**Check Your Progress**

**Hockey** The Brownsville Badgers hockey team has 14 members. Two members of the team are to be selected to be the team’s co-captains. How many different pairs of players can be selected to be the co-captains?
**EXAMPLES**

**INTRODUCTIONS**

Ten managers attend a business meeting. Each person exchanges names with each other person once. How many introductions will there be?

There are $10 \cdot 9$ ways to choose 2 people.

There are $2 \cdot 1$ ways to arrange the 2 people.

$$\frac{10 \cdot 9}{2 \cdot 1} = \frac{90}{2} = 45$$

There are 45 introductions.

If the introductions in Example 2 are made at random, what is the probability that Ms. Apple and Mr. Zimmer will be the last managers to exchange names?

Since there are 45 introductions and only one favorable outcome, the probability that Ms. Apple and Mr. Zimmer will be the last managers to exchange names is $\frac{1}{45}$.

**Check Your Progress**

a. **INTRODUCTIONS**

Fifteen managers attend a business meeting. Each person exchanges names with each other person once. How many introductions will there be?

105

b. What is the probability that Ms. Apple and Mr. Zimmer will be the last managers to exchange names?

$\frac{1}{105}$
EXAMPLE Solve Using the Act It Out Strategy

LUNCH Salvador is looking for his lunch money, which he put in one of the pockets of his backpack this morning. If the backpack has six pockets, what is the probability that he will find the money in the first pocket that he checks?

UNDERSTAND You know that there are six pockets in Salvador’s backpack and that one of the pockets contains his lunch money.

PLAN Toss a number cube several times. If the cube lands on 1, Salvador will find the money in the first pocket that he checks. If the cube lands on 2, 3, 4, 5, or 6, Salvador will not find the money in the first pocket that he checks.

SOLVE Toss the cube and make a table of the results.

<table>
<thead>
<tr>
<th>Trials</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcome</td>
<td>4</td>
<td>5</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

The highlighted entries show that 2 out of the 12 trials resulted in Salvador finding his lunch money in the first pocket that he checks.

So, the probability is \( \frac{2}{12} \) or \( \frac{1}{6} \).

CHECK Repeat the experiment several times to see whether the results agree.

Check Your Progress PHOTOGRAPHS A photographer is taking a picture of the four members in Margaret’s family. Margaret’s grandmother will stand on the right. How many different ways can the photographer arrange the family members in a row for the photo?
**Main Idea**

- Find and compare experimental and theoretical probabilities.

**Build Your Vocabulary** (pages 202–203)

**Experimental probability** is based on what actually occurred during an experiment. **Theoretical probability** is based on what should happen when conducting an experiment.

**Example: Experimental Probability**

A spinner is spun 50 times, and it lands on the color blue 15 times. What is the experimental probability of spinning blue?

\[ P(\text{blue}) = \frac{\text{number of times blue is spun}}{\text{number of possible outcomes}} = \frac{15}{50} = \frac{3}{10} \]

The experimental probability of spinning the color blue is \( \frac{3}{10} \).

**Check Your Progress**

A marble is pulled from a bag of colored marbles 30 times and 18 of the pulls result in a yellow marble. What is the experimental probability of pulling a yellow marble?

\[ \frac{3}{5} \]
Experimental and Theoretical Probability

The graph shows the results of an experiment in which a number cube is rolled 30 times.

1. Find the experimental probability of rolling a 5.

\[
P(5) = \frac{\text{number of times 5 occurs}}{\text{number of possible outcomes}} = \frac{4}{30} \quad \text{or} \quad \frac{2}{15}
\]

The experimental probability of rolling a 5 is \(\frac{2}{15}\).

2. Compare the experimental probability of rolling a 5 to its theoretical probability.

The theoretical probability of rolling a 5 on a number cube is \(\frac{1}{6}\). So, the theoretical probability is close to the experimental probability of \(\frac{2}{15}\).

Check Your Progress

The graph shows the result of an experiment in which a coin was tossed 150 times.

a. Find the experimental probability of tossing heads for this experiment.

\[
\frac{8}{15}
\]

b. Compare the experimental probability of tossing heads to its theoretical probability.

The theoretical probability of tossing heads is \(\frac{1}{2}\). So, the theoretical probability is close to the experimental probability of \(\frac{8}{15}\).
9–8  Compound Events

**Main Idea**
- Find the probability of independent and dependent events.

**Key Concept**

**Probability of Two Independent Events**

The probability of two independent events can be found by multiplying the probability of the first event by the probability of the second event.

**Example**

1. **LUNCH**
   For lunch, Jessica may choose from a turkey sandwich, a tuna sandwich, a salad, or a soup. For a drink, she can choose juice, milk, or water. If she chooses a lunch and a drink at random, what is the probability that she chooses a sandwich (of either kind) and juice? (parallels Example 2 in text)

   \[
   P(\text{sandwich}) = \frac{1}{2} \quad \quad P(\text{juice}) = \frac{1}{3}
   \]

   \[
   P(\text{sandwich and juice}) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6} \quad \text{or} \quad P(\text{sandwich}) \cdot P(\text{juice}) = \frac{1}{6}
   \]

   So, the probability that she chooses a sandwich and juice is \(\frac{1}{6}\).

**Check Your Progress**

1. **SWEATS**
   Zachary has a blue, a red, a gray, and a white sweatshirt. He also has blue, red, and gray sweatpants. If Zachary randomly pulls a sweatshirt and a pair of sweatpants from his drawer, what is the probability that they will both be blue?

   \[
   \frac{1}{12}
   \]

**Build Your Vocabulary**

- If one event affects the outcome of a second event, the events are called **dependent events**.
- If two events cannot happen at the same time, then they are **disjoint events**.
**EXAMPLES**

### Dependent Events

1. **SOCKS** There are 4 black, 6 white, and 2 blue socks in a drawer. José randomly selects two socks without replacing the first sock. What is the probability that he selects two white socks? (parallels Example 3 in text)

   \[
P(\text{first sock is white}) = \frac{6}{12} \quad \text{There are 6 white socks and 12 total socks.}
   \]

   \[
P(\text{second sock is white}) = \frac{5}{11} \quad \text{After one white sock is removed, there are 5 white socks and 11 total socks.}
   \]

   \[
P(\text{two white socks}) = \frac{6}{12} \cdot \frac{5}{11} \quad \text{or} \quad \frac{5}{22}
   \]

2. **Disjoint Events**

   **MONTHS** A month of the year is randomly selected. What is the probability of the month ending in the letter \(Y\) or the letter \(R\)? (parallels Example 4 in text)

   They are disjoint events since it is impossible to have a month ending in both the letter \(Y\) and the letter \(R\).

   \[
P(\text{ending in } Y \text{ or } R) = \frac{8}{12} \quad \text{There are 8 months that end in } Y \text{ or } R.
   \]

   \[
   \text{There are 12 months.}
   \]

### Check Your Progress

a. **GAMES** Janet has a card game that uses a deck of 48 cards – 16 red, 16 blue, and 16 green. If she randomly selects two cards without replacing the first, what is the probability that both are green?

   \[
   \frac{5}{47} \quad \text{or about 10.6%}
   \]

b. **MARBLES** There are 12 yellow, 3 black, 5 red, and 8 blue marbles in a bag. Joseph randomly selects one marble from the bag. What is the probability that the marble selected will be black or red?

   \[
   \frac{2}{7}
   \]
### BRINGING IT ALL TOGETHER

**STUDY GUIDE**

<table>
<thead>
<tr>
<th>FOLDABLES</th>
<th>VOCABULARY PUZZLEMAKER</th>
<th>BUILD YOUR VOCABULARY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use your Chapter 9 Foldable to help you study for your chapter test.</td>
<td>To make a crossword puzzle, word search, or jumble puzzle of the vocabulary words in Chapter 9, go to: glencoe.com</td>
<td>You can use your completed Vocabulary Builder (pages 202–203) to help you solve the puzzle.</td>
</tr>
</tbody>
</table>

#### 9-1 Simple Events

For Questions 1–3, a bag contains 4 green, 6 orange, and 10 purple blocks. Find each probability if you draw one block at random from the bag. Write as a fraction in simplest form.

1. \( P(\text{green}) \)  
2. \( P(\text{orange}) \)  
3. \( P(\text{purple}) \)

<table>
<thead>
<tr>
<th>Event</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>green</td>
<td>( \frac{1}{5} )</td>
</tr>
<tr>
<td>orange</td>
<td>( \frac{3}{10} )</td>
</tr>
<tr>
<td>purple</td>
<td>( \frac{1}{2} )</td>
</tr>
</tbody>
</table>

#### 9-2 Sample Spaces

4. **PHONES** A phone company offers three different calling features (caller ID, call waiting, and call forward) and two different calling plans (Plan A or Plan B). Find the sample space for all possibilities of a calling feature and a calling plan.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>caller ID</td>
<td>plan A</td>
</tr>
<tr>
<td>caller ID</td>
<td>plan B</td>
</tr>
<tr>
<td>call waiting</td>
<td>plan A</td>
</tr>
<tr>
<td>call waiting</td>
<td>plan B</td>
</tr>
<tr>
<td>call forward</td>
<td>plan A</td>
</tr>
<tr>
<td>call forward</td>
<td>plan B</td>
</tr>
</tbody>
</table>
Chapter 9

9-3  The Fundamental Counting Principle

5. Underline the correct term to complete the sentence: The operation used in the Fundamental Counting Principle is (addition, multiplication).

Use the Fundamental Counting Principle to find the total number of outcomes in each situation.

6. Tossing a coin and rolling a 6-sided number cube.

   12

7. Making a sandwich using whole wheat or sourdough bread, ham or turkey, and either cheddar, swiss, or provolone cheese.

   12

8. Choosing a marble from a bag containing 10 differently colored marbles and spinning the spinner at the right.

   40

9-4  Permutations

9. LETTERS How many permutations are there of the letters in the word pizza?

   120

10. BASEBALL In how many ways can the six infielders of a baseball team stand in a row for autograph signing?

    720

11. NUMBERS How many 4-digit passwords can be formed using the digits 1, 3, 4, 5, 7, and 9? Assume no number can be used more than once.

    360
Complete each sentence.

12. You can find the number of combinations of objects in a set by dividing the number of permutations of the entire set by the number of ways each smaller set can be arranged.

13. A combination is an arrangement or listing in which order is not important.

14. The burger shop offers 3 choices of condiments from the following: lettuce, onions, pickles, ketchup, and mustard. How many different combinations of condiments can you have on your burger?

10

15. TRAVEL Four friends are driving to the beach. In how many different ways can two friends sit in the front and two friends sit in the back if Raul must be the driver?

6

Underline the correct term(s) to complete each sentence.

16. The word experimental means based on (experience, theory).

17. Theoretical probability is based on what (you actually try, is expected).

18. (Experimental, theoretical) probability can be based on past performance and can be used to make predictions about future events.
Sue has 5 different kinds of shoes: sneakers, sandals, boots, moccasins, and heels.

19. If she chooses a pair each day for two weeks, and chooses moccasins 8 times, what is the experimental probability that moccasins are chosen?

\[
\frac{4}{7}
\]

20. Find the theoretical probability of choosing the moccasins.

\[
\frac{1}{5}
\]

### Compound Events

State whether each sentence is true or false. If false, replace the underlined word to make the sentence true.

21. A compound event consists of more than one single event.

**True**

22. When the outcome of the first event does not have any effect on the second event it is called a simple event.

**False; independent**

23. A yellow and a green cube are rolled. What is the probability that an even number is rolled on the yellow cube and a number less than 3 is rolled on the green cube?

\[
\frac{1}{6}
\]

24. There are 4 chocolate chip, 6 peanut butter, and 2 sugar cookies in a box. Malena randomly selects two cookies without replacing the first. Find the probability that she selects a peanut butter cookie and then a sugar cookie.

\[
\frac{1}{11}
\]
Check the one that applies. Suggestions to help you study are given with each item.

I completed the review of all or most lessons without using my notes or asking for help.
- You are probably ready for the Chapter Test.
- You may want to take the Chapter 9 Practice Test on page 503 of your textbook as a final check.

I used my Foldable or Study Notebook to complete the review of all or most lessons.
- You should complete the Chapter 9 Study Guide and Review on pages 498–502 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may want to take the Chapter 9 Practice Test on page 503 of your textbook.

I asked for help from someone else to complete the review of all or most lessons.
- You should review the examples and concepts in your Study Notebook and Chapter 9 Foldable.
- Then complete the Chapter 9 Study Guide and Review on pages 498–502 of your textbook.
- If you are unsure of any concepts or skills, refer back to the specific lesson(s).
- You may also want to take the Chapter 9 Practice Test on page 503 of your textbook.

Student Signature

Parent/Guardian Signature

Teacher Signature
Geometry: Polygons

Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with a sheet of 11” by 17” paper.

**STEP 1** Fold a 2” tab along the long side of the paper.

**STEP 2** Unfold the paper and fold in thirds widthwise.

**STEP 3** Open and draw lines along the folds. Label the head of each column as shown. Label the front of the folded table with the chapter title.

**NOTE-TAKING TIP:** As you study a chapter, take notes, record concepts, and write examples about important definitions and concepts.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 10. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>acute triangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>adjacent angles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>complementary angles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>congruent angles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>congruent segments</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>congruent segments</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>equilateral [EH-kwuh-LA-tuh-rull] triangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>indirect measurement</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>isosceles [y-SAHS-LEEZ] triangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>line symmetry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>obtuse triangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>parallelogram</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vocabulary Term</td>
<td>Found on Page</td>
<td>Definition</td>
<td>Description or Example</td>
</tr>
<tr>
<td>----------------------------------</td>
<td>---------------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>quadrilateral [KWAH-druh-LA-tuh-ruhl]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>reflection</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rhombus [RAHM-buhs]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>scalene [SKAY-LEEN] triangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>similar figures</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>straight angle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>supplementary angles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tessellation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>translation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>trapezoid [TRA-puh-ZOYD]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vertex</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vertical angles</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
An angle has two sides that share a common endpoint and is measured in units called degrees. The point where the sides of an angle meet is called the vertex.

**EXAMPLE**

**Naming Angles**

Name the angle at the right.

- Use the vertex as the middle letter and a point from each side.
  \[ \angle FGH \text{ or } \angle HGF \]

- Use the vertex only.
  \[ \angle G \]

- Use a number.
  \[ \angle 2 \]

The angle can be named in four ways:

\[ \angle FGH, \angle HGF, \angle G, \text{ or } \angle 2 \]

**Check Your Progress**

Name the angle below.

\[ \angle QRS, \angle SRQ, \angle R, \text{ or } \angle 3 \]
**Remember It**

A ray starts at a point and goes without end in one direction.

**Build Your Vocabulary** (pages 225–226)

- A right angle measures exactly 90°.
- An acute angle measures less than 90°.
- An obtuse angle measures between 90° and 180°.
- A straight angle measures exactly 180°.

**Examples**

Classify Angles

Classify each angle as acute, obtuse, right, or straight.

1. 

The angle is exactly 180°, so it is a **straight** angle.

2. 

The angle is less than 90°, so it is an **acute** angle.

**Check Your Progress**

Classify each angle as acute, obtuse, right, or straight.

a. 

right

b. 

obtuse
Two angles that have the same measure are congruent.

Two angles are vertical if they are opposite angles formed by the intersection of two lines.

Two angles are adjacent if they share a common vertex, a common side, and do not overlap.

**Example**

Determine if each pair of angles in the figure at the right are vertical angles, adjacent angles, or neither.

a. $\angle 3$ and $\angle 5$

Since $\angle 3$ and $\angle 5$ are opposite angles formed by the intersection of two lines, they are vertical angles.

b. $\angle 3$ and $\angle 4$

$\angle 3$ and $\angle 4$ share a common vertex and side, and do not overlap. So, they are adjacent angles.

c. $\angle 4$ and $\angle 5$

$\angle 4$ and $\angle 5$ share a common vertex and side, and do not overlap. So, they are adjacent angles.

**Check Your Progress**

Determine if each pair of angles in the figure at the right are vertical angles, adjacent angles, or neither.

a. $\angle 1$ and $\angle 2$

adjacent angles

b. $\angle 2$ and $\angle 5$

vertical angles

c. $\angle 1$ and $\angle 4$

neither
Main Idea
Identify complementary and supplementary angles and find missing angle measures.

Build Your Vocabulary (pages 225–226)

Complementary angles have a sum of $90^\circ$.
Supplementary angles have a sum of $180^\circ$.

Examples
Classify Angles
Classify each pair of angles as complementary, supplementary, or neither.

1. $128^\circ + 52^\circ = 180^\circ$
   So, the angles are supplementary.

2. $\angle x$ and $\angle y$ form a right angle.
   So, the angles are complementary.

Check Your Progress
Classify each pair of angles as complementary, supplementary, or neither.

a. Complementary

Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.
**EXAMPLE**  
**Find a Missing Angle Measure**

Angles $PQS$ and $RQS$ are supplementary. If $m\angle PQS = 56^\circ$, find $m\angle RQS$.

Since $\angle PQS$ and $\angle RQS$ are supplementary, 

$m\angle PQS + m\angle RQS = 180^\circ$.

$m\angle PQS + m\angle RQS = 180$  
Write the equation.

$56 + m\angle RQS = 180$  
Replace $m\angle PQS$ with 56.

$-56 \quad -56$  
Subtract 56 from each side.

$m\angle RQS = 124$  
$180 - 56 = 124$

The measure of $\angle RQS$ is $124^\circ$.

**Check Your Progress**  
Angles $MNP$ and $KNP$ are complementary. If $m\angle MNP = 23^\circ$, find $m\angle KNP$. 

**Homework Assignment**

Page(s):  
Exercises:
A graph that shows data as parts of a whole is a circle graph.

**EXAMPLE Display Data in a Circle Graph**

**SPORTS** In a survey, a group of middle school students were asked to name their favorite sport. The results are shown in the table. Make a circle graph of the data.

<table>
<thead>
<tr>
<th>Sport</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>football</td>
<td>30%</td>
</tr>
<tr>
<td>basketball</td>
<td>25%</td>
</tr>
<tr>
<td>baseball</td>
<td>22%</td>
</tr>
<tr>
<td>tennis</td>
<td>8%</td>
</tr>
<tr>
<td>other</td>
<td>15%</td>
</tr>
</tbody>
</table>

- Find the degrees for each part. Round to the nearest whole degree.

  - football: \(30\% \text{ of } 360^\circ = 0.30 \times 360^\circ = 108^\circ\)
  - basketball: \(25\% \text{ of } 360^\circ = 0.25 \times 360^\circ = 90^\circ\)
  - baseball: \(22\% \text{ of } 360^\circ = 0.22 \times 360^\circ \approx 79^\circ\)
  - tennis: \(8\% \text{ of } 360^\circ = 0.08 \times 360^\circ \approx 29^\circ\)
  - other: \(15\% \text{ of } 360^\circ = 0.15 \times 360^\circ \approx 54^\circ\)

- Draw a circle with a radius marked as shown. Then use a protractor to draw the first angle, in this case \(108^\circ\). Repeat this step for each section.

**WRITE IT**

Write a proportion to convert 65\% to the number of degrees in a part of a circle graph.
Label each section of the graph with the category and percent. Give the graph a title.

EXAMPLE

Construct a Circle Graph

MOVIES Gina has the following types of movies in her DVD collection. Make a circle graph of the data.

<table>
<thead>
<tr>
<th>Type of Movie</th>
<th>Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>action</td>
<td>24</td>
</tr>
<tr>
<td>comedy</td>
<td>15</td>
</tr>
<tr>
<td>science fiction</td>
<td>7</td>
</tr>
</tbody>
</table>

- Find the total number of DVDs: $24 + 15 + 7 = 46$.

- Find the ratio that compares each number with the total. Write the ratio as a decimal number rounded to the nearest hundredth.

  - action: $\frac{24}{46} \approx 0.52$
  - comedy: $\frac{15}{46} \approx 0.33$
  - science fiction: $\frac{7}{46} \approx 0.15$

(continued on the next page)
• Find the number of degrees for each section of the graph.

- action: \[0.52 \times 360° = 187°\]
- comedy: \[0.33 \times 360° = 119°\]
- science: \[0.15 \times 360° = 54°\]

• Draw the circle graph.

Check Your Progress

**a. ICE CREAM** In a survey, a group of students were asked to name their favorite flavor of ice cream. The results are shown in the table. Make a circle graph of the data.

<table>
<thead>
<tr>
<th>Flavor</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>chocolate</td>
<td>30%</td>
</tr>
<tr>
<td>cookie dough</td>
<td>25%</td>
</tr>
<tr>
<td>peanut butter</td>
<td>15%</td>
</tr>
<tr>
<td>strawberry</td>
<td>10%</td>
</tr>
<tr>
<td>other</td>
<td>20%</td>
</tr>
</tbody>
</table>

**b. MARBLES** Michael has the following colors of marbles in his marble collection. Make a circle graph of the data.

<table>
<thead>
<tr>
<th>Color</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>black</td>
<td>12</td>
</tr>
<tr>
<td>green</td>
<td>9</td>
</tr>
<tr>
<td>red</td>
<td>5</td>
</tr>
<tr>
<td>gold</td>
<td>3</td>
</tr>
</tbody>
</table>
EXAMPLES

Analyze a Circle Graph

**VOTING** The circle graph below shows the percent of voters in a town who are registered with a political party.

Which party has the most registered voters?
The largest section of the circle is the one representing **Democrats**. So, the Democratic party has the most registered voters.

If the town has 3,400 registered Republicans, about how many voters are registered in all?
Republicans: 42% of registered voters = 3,400

\[
0.42 \times n = 3,400
\]

\[
0.42n = 3,400
\]

\[
n \approx 8,095
\]

So, there are about **8,095** registered voters in all.

**Check Your Progress**  
SPORTS The circle graph below shows the responses of middle school students to the question “Should teens be allowed to play professional sports?”

a. Which response was the greatest?

b. If there were 1,500 middle school students, how many had no opinion?

**HOMEWORK ASSIGNMENT**

Page(s):
Exercises:

Math Connects, Course 2  235
A triangle is a figure with three sides and three angles. Sides with the same length are congruent segments.

**Example:** Find a Missing Measure

**ALGEBRA** Find $m\angle A$ in $\triangle ABC$ if $m\angle A = m\angle B$, and $m\angle C = 80^\circ$.

Since the sum of the angle measures in a triangle is $180^\circ$, 

$$m\angle A + m\angle B + m\angle C = 180^\circ.$$ 

Let $x$ represent $m\angle A$. Since $m\angle A = m\angle B$, $x$ also represents $m\angle B$.

$$x + x + 80 = 180 \quad \text{Write the equation.}$$

$$2x + 80 = 180 \quad x + x = 2x$$

$$2x = 100 \quad \text{Subtract 80 from each side.}$$

$$x = 50 \quad \text{Divide each side by 2.}$$

So, $m\angle A = 50^\circ$.

**Check Your Progress** **ALGEBRA** Find $m\angle M$ in $\triangle MNO$ if $m\angle N = 75^\circ$ and $m\angle O = 67^\circ$.

$38^\circ$
An **acute triangle** has all acute angles. A **right triangle** has one right angle. An **obtuse triangle** has one obtuse angle.

A **scalene triangle** has no congruent sides. An **isosceles triangle** has at least 2 congruent sides. An **equilateral triangle** has three congruent sides.

---

**EXAMPLE**

**TEST EXAMPLE** An airplane has wings that are shaped like triangles. What is the missing measure of the angle?

A 41°  B 31°  C 26°  D 21°

**Read the Item**

To find the missing measure, write and solve an equation.

**Solve the Item**

\[ x + 47 + 112 = 180 \]

The sum of the measures is 180.

\[ x + 159 = 180 \]

Simplify.

\[ -159 \]

Subtract 159 from each side.

\[ x = 21 \]

The missing measure is 21°. The answer is D.

---

**Check Your Progress**

**MULTIPLE CHOICE** A piece of fabric is shaped like a triangle. Find the missing angle measure.

F 73°  
G 49°  
H 58°  
J 53°

G
**EXAMPLE  Solve Using Logical Reasoning**

**GEOMETRY** Draw an equilateral triangle. How can you confirm that it is equilateral?

**UNDERSTAND** You know that equilateral triangles have three congruent sides. You need to confirm whether or not a drawn triangle is equilateral.

**PLAN** Draw an equilateral triangle. Measure the sides to confirm that all three sides are congruent.

**SOLVE** Draw the triangle.

![Equilateral Triangle]

Measure the sides using a ruler or centimeter ruler. The side lengths are 2.6 centimeters, 2.6 centimeters, and 2.6 centimeters. Since all three sides are congruent, the triangle is equilateral.

**CHECK** Since all three sides are congruent, the triangle is equilateral. You can have someone else also measure the sides to check that the triangle is equilateral.

**Check Your Progress** **GEOMETRY** Do the angles in an equilateral triangle have a special relationship?

Yes, all three angles are congruent.
A quadrilateral is a closed figure with four sides and four angles.

A parallelogram is a quadrilateral with opposite sides parallel and opposite sides congruent.

A trapezoid is a quadrilateral with one pair of parallel sides.

A rhombus is a parallelogram with four congruent sides.

**Examples**

Classify Quadrilaterals

1. The quadrilateral has 4 right angles and opposite sides are congruent. It is a rectangle.

2. The quadrilateral has one pair of parallel sides. It is a trapezoid.
Check Your Progress  Classify the quadrilateral using the name that best describes it.

a. parallelogram

b. square

EXAMPLE  Find a Missing Measure

1 ALGEBRA Find the value of \(x\) in the quadrilateral shown.

Write and solve an equation. Let \(x\) represent the missing measure.

\[
60 + 120 + 60 + x = 360
\]

The sum of the measures is 360°.

\[
240 + x = 360
\]

Simplify.

\[
-240
\]

Subtract 240 from both sides.

\[
x = 120
\]

So, the missing angle measure is 120°.

Check Your Progress  Find the missing angle measure in the quadrilateral.

134°
**Similar Figures**

**Main Idea**
- Determine whether figures are similar and find a missing length in a pair of similar figures.

**Key Concept**

**Similar Figures** If two figures are similar, then
- the corresponding sides are proportional, and
- the corresponding angles are congruent.

**Example** Identify Similar Figures

Which rectangle below is similar to rectangle $FGHI$?

![Rectangles Diagram]

Compare the ratios of the corresponding sides.

<table>
<thead>
<tr>
<th>Rectangle</th>
<th>$FG$</th>
<th>$LM$</th>
<th>$AB$</th>
<th>$BC$</th>
<th>$QR$</th>
<th>$RS$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$FG$</td>
<td>9 ft</td>
<td>8 ft</td>
<td>9 ft</td>
<td>6 ft</td>
<td>9 ft</td>
<td>12 ft</td>
</tr>
<tr>
<td>$LM$</td>
<td>2 ft</td>
<td>2 ft</td>
<td>2 ft</td>
<td>2 ft</td>
<td>2 ft</td>
<td>6 ft</td>
</tr>
<tr>
<td>$AB$</td>
<td>6 ft</td>
<td>6 ft</td>
<td>6 ft</td>
<td>6 ft</td>
<td>6 ft</td>
<td>6 ft</td>
</tr>
<tr>
<td>$BC$</td>
<td>2 ft</td>
<td>2 ft</td>
<td>2 ft</td>
<td>2 ft</td>
<td>2 ft</td>
<td>6 ft</td>
</tr>
<tr>
<td>$QR$</td>
<td>12 ft</td>
<td>12 ft</td>
<td>12 ft</td>
<td>12 ft</td>
<td>12 ft</td>
<td>12 ft</td>
</tr>
<tr>
<td>$RS$</td>
<td>6 ft</td>
<td>6 ft</td>
<td>6 ft</td>
<td>6 ft</td>
<td>6 ft</td>
<td>6 ft</td>
</tr>
</tbody>
</table>

- $FG/LM = 9/8$
- $FG/AB = 9/6$
- $FG/QR = 9/12$
- $GH/MN = 3/2$
- $GH/BC = 3/2$
- $GH/RS = 3/6$

Not similar  Similar  Not similar

So, rectangle $FGHI$ is similar to rectangle $ABCD$. 

### BUILD YOUR VOCABULARY (pages 225–226)

Figures that have the same shape but not necessarily the same size are similar figures.

The sides of similar figures that “match” are corresponding sides.

The angles of similar figures that “match” are corresponding angles.
Check Your Progress Which rectangle from Example 1 is similar to rectangle WXYZ shown?

rectangle QRST

BUILD YOUR VOCABULARY (pages 225–226)

Indirect measurement uses similar figures to find the length, width, or height of objects that are too difficult to measure directly.

EXAMPLE

ARCHITECTURE A rectangular picture window 12 feet long and 6 feet wide needs to be shortened to 9 feet in length to fit a redesigned wall. If the architect wants the new window to be similar to the old window, how wide will the new window be? (parallels Example 3 in text)

\[
\frac{12}{9} = \frac{6}{w}
\]

Write a proportion.

\[
12w = 9(6)
\]

Find the cross products.

\[
12w = 54
\]

Simplify.

\[
w = 4.5
\]

Divide each side by 12.

So, the width of the new window will be 4.5 feet.

Check Your Progress Tom has a rectangular garden that has a length of 12 feet and a width of 8 feet. He wishes to start a second garden that is similar to the first and will have a width of 6 feet. Find the length of the new garden.

9 ft
**Main Idea**

- Classify polygons and determine which polygons can form a tessellation.

**Build Your Vocabulary** (pages 225–226)

A *polygon* is a simple, closed figure formed by three or more straight line segments.

A *regular polygon* has all sides congruent and all angles congruent.

A polygon is named by the number of sides it has:
- **pentagon** (5 sides),
- **hexagon** (6 sides),
- **heptagon** (7 sides),
- **octagon** (8 sides),
- **nonagon** (9 sides), and
- **decagon** (10 sides).

**Examples**

**Classify Polygons**

Determine whether each figure is a polygon.

1. The figure is not a polygon since it has a **curved** side.

2. This figure has 6 sides that are not all of equal length. It is a **hexagon** that is not **regular**.

**Check Your Progress**

Determine whether each figure is a polygon. If it is, classify the polygon and state whether it is regular. If it is not a polygon, explain why.

a. **pentagon, not regular**

b. **not a polygon, sides overlap**
A repetitive pattern of polygons that fit together with no overlaps or holes is called a tessellation.

**EXAMPLE**  **Tessellations**  (parallels Example 4 in text)

1. **PATTERNS**  Ms. Pena is creating a pattern on her wall. She wants to use regular hexagons. Can Ms. Pena make a tessellation with regular hexagons?

The measure of each angle in a regular hexagon is $120^\circ$.

The sum of the measures of the angles where the vertices meet must be $360^\circ$.

So, solve $120n = 360$.

\[
120n = 360 \quad \text{Write the equation.}
\]

\[
\frac{120n}{120} = \frac{360}{120} \quad \text{Divide each side by } 120.
\]

\[
n = 3
\]

Since $120^\circ$ divides evenly into $360^\circ$, the sum of the measures where the vertices meet is $360^\circ$. So, Ms. Pena can make a tessellation with regular hexagons.

**Check Your Progress**  **QUILTING**  Emily is making a quilt using fabric pieces shaped as equilateral triangles. Can Emily tessellate the quilt with these fabric pieces?

Yes, they can be arranged in a way that the angles where the vertices meet make $360^\circ$. 
**Main Idea**

- Graph translations of polygons on a coordinate plane.

**Build Your Vocabulary** (pages 225–226)

A transformation maps one figure onto another.

A translation is a transformation where a figure is moved without turning it.

The original figure and the translated figure are congruent figures.

**Example** Graph a Translation

1. Translate \( \triangle ABC \) 5 units left and 1 unit up.

   - Move each vertex of the figure 5 units left and 1 unit up. Label the new vertices \( A', B', \) and \( C' \).
   - Connect the vertices to draw the triangle. The coordinates of the vertices of the new figure are
     
     \[
     A'(-4, -1), \quad B'(-1, 2), \quad C'(0, -1)
     \]

   **Check Your Progress**

   Translate \( \triangle DEF \) 3 units left and 2 units down.

   **Remember It**

   The order of a translation of a figure does not matter. Moving a figure to the side \( x \) units and then up \( y \) units is the same as moving it up \( y \) units and then to the side \( x \) units.
EXAMPLE

Find Coordinates of a Translation

Trapezoid $GHIJ$ has vertices $G(-4, 1)$, $H(-4, 3)$, $I(-2, 3)$, and $J(-1, 1)$. Find the vertices of trapezoid $G'H'I'J'$ after a translation of 5 units right and 3 units down. Then graph the figure and its translated image.

Add 5 to each $x$-coordinate. Add -3 to each $y$-coordinate.

<table>
<thead>
<tr>
<th>Vertices of Trapezoid $GHIJ$</th>
<th>$(x + 5, y - 3)$</th>
<th>Vertices of Trapezoid $G'H'I'J'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G(-4, 1)$</td>
<td>$(-4 + 5, 1 - 3)$</td>
<td>$G'(1, -2)$</td>
</tr>
<tr>
<td>$H(-4, 3)$</td>
<td>$(-4 + 5, 3 - 3)$</td>
<td>$H'(1, 0)$</td>
</tr>
<tr>
<td>$I(-2, 3)$</td>
<td>$(-2 + 5, 3 - 3)$</td>
<td>$I'(3, 0)$</td>
</tr>
<tr>
<td>$J(-1, 1)$</td>
<td>$(-1 + 5, 1 - 3)$</td>
<td>$J'(4, -2)$</td>
</tr>
</tbody>
</table>

The coordinates of trapezoid $G'H'I'J'$ are $G'(1, -2)$, $H'(1, 0)$, $I'(3, 0)$, and $J'(4, -2)$.

Check Your Progress  

Triangle $MNO$ has vertices $M(-5, -3)$, $N(-7, 0)$, and $O(-2, 3)$. Find the vertices of triangle $M'N'O'$ after a translation of 6 units right and 3 units up. Then graph the figure and its translated image.

**Homework Assignment**

Page(s):

Exercises:
Figures that match exactly when they are folded in half have line symmetry.

Each fold line is called a line of symmetry.

**EXAMPLES**

**Identify Lines of Symmetry**

**LETTERS** Determine whether each letter has a line of symmetry. If so, copy the figure and draw all lines of symmetry.

1. This figure has line symmetry.
   There are two lines of symmetry.

2. This figure has line symmetry.
   There is one line of symmetry.

3. This figure does not have line symmetry.

**Check Your Progress** Determine whether each figure has line symmetry. If so, copy the figure and draw all lines of symmetry.

a. 

b. 

---

*Math Connects, Course 2* 247
A reflection is a mirror image of the original figure that is the result of a transformation over a line called a line of reflection.

**Reflect a Figure Over the x-axis**

Quadrilateral $QRST$ has vertices $Q(-1, 1), R(0, 3), S(3, 2),$ and $T(4, 0)$. Graph the figure and its reflected image over the $x$-axis. Then find the coordinates of the reflected image.

The $x$-axis is the line of reflection. So, plot each vertex of $QR'S'T'$ the same distance from the $x$-axis as its corresponding vertex on $QRST$.

$Q'(-1, -1) \quad R'(0, -3) \quad S'(3, -2) \quad T'(4, 0)$

**Check Your Progress** Quadrilateral $ABCD$ has vertices $A(-3, 2), B(-1, 5), C(3, 3),$ and $D(2, 1)$. Graph the figure and its reflection over the $x$-axis. Then find the coordinates of the reflected image.

$A'(-3, -2), B'(-1, -5), C'(3, -3), D'(2, -1)$
Reflect a Figure over the \( y \)-axis

Triangle \( XYZ \) has vertices \( X(1, 2) \), \( Y(2, 1) \), and \( Z(1, -2) \). Graph the figure and its reflected image over the \( y \)-axis. Then find the coordinates of the reflected image.

The \( y \)-axis is the line of reflection. So, plot each vertex of \( XYZ' \) the same distance from the \( y \)-axis and its corresponding vertex on \( XYZ \).

\[
\begin{align*}
X'(−1, 2) \\
Y'(−2, 1) \\
Z'(−1, −2)
\end{align*}
\]

Check Your Progress  Triangle \( QRS \) has vertices \( Q(3, 4) \), \( R(1, 0) \), and \( S(6, 2) \). Graph the figure and its reflection over the \( y \)-axis. Then find the coordinates of the reflected image.

\[
\begin{align*}
Q'(-3, 4), R'(-1, 0) \\
S'(-6, 2)
\end{align*}
\]
## Angle Relationships

Classify each angle as **acute**, **obtuse**, or **right**.

1. [Image of an angle]
   - **acute**

2. [Image of an angle]
   - **right**

3. [Image of an angle]
   - **obtuse**

## Complementary and Supplementary Angles

Complete each sentence.

4. The sum of the measures of **supplementary** angles is $180^\circ$.

5. The sum of the measures of **complementary** angles is $90^\circ$.

6. If $\angle A$ and $\angle B$ are supplementary angles and $m\angle B = 43^\circ$, find $m\angle A$.
   - $137^\circ$

## Statistics: Display Data in a Circle Graph

Find the number of degrees for each part of the graph at the right.

7. A [90]
8. B [45]
9. C [135]
10-4 Triangles

Complete the table to help you remember the ways to classify triangles.

<table>
<thead>
<tr>
<th>Type of Triangle</th>
<th>Classified by Angles or Sides</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>10. acute</td>
<td>angles</td>
<td>all acute angles</td>
</tr>
<tr>
<td>11. obtuse</td>
<td>angles</td>
<td>1 obtuse angle</td>
</tr>
<tr>
<td>12. scalene</td>
<td>sides</td>
<td>no congruent sides</td>
</tr>
<tr>
<td>13. right</td>
<td>angles</td>
<td>1 right angle</td>
</tr>
<tr>
<td>14. equilateral</td>
<td>sides</td>
<td>3 congruent sides</td>
</tr>
</tbody>
</table>

10-5 Problem-Solving Investigation: Logical Reasoning

15. RACES Marcus, Elena, Pedro, Keith, and Darcy ran a 2-mile race. Darcy finished directly after Pedro, Elena finished before Marcus, and Keith finished first. If Pedro finished third, order the runners from first to last.

Keith, Elena, Pedro, Darcy, Marcus

10-6 Quadrilaterals

Find the value of $x$ in the quadrilateral.

16. $150^\circ$, $75^\circ$, $58^\circ$, $x$

17. $x^\circ$, $45^\circ$

77

135
10-7

Similar Figures

18. Find the value of $x$ if $\triangle ABC \sim \triangle DEF$.

10-8

Polygons and Tessellations

Underline the correct term to complete each sentence.

19. A polygon can have (two, three) or more straight lines.

20. To find the sum of the angle measures in a regular polygon, draw all the diagonals from one vertex, count the number of (angles, triangles) formed, and multiply by 180°.

10-9

Translations

21. Triangle $ABC$ with vertices $A(2, 4), B(-4, 6), \text{ and } C(1, -5)$ is translated 2 units right and 3 units down. What are the coordinates of $B$?

$(−2, 3)$

10-10

Reflections

Underline the correct word(s) to complete the sentence.

22. The image of a reflection is (larger than, the same size as) the original figure.

23. Triangle $DEF$ has vertices $D(-5, 2), E(-4, -2), \text{ and } F(-3, 0)$. It is reflected over the $y$-axis. What are the coordinates of $D$?

$(5, 2)$
ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
  • You are probably ready for the Chapter Test.
  • You may want to take the Chapter 10 Practice Test on page 567 of your textbook as a final check.

☐ I used my Foldable or Study Notebook to complete the review of all or most lessons.
  • You should complete the Chapter 10 Study Guide and Review on pages 563–566 of your textbook.
  • If you are unsure of any concepts or skills, refer to the specific lesson(s).
  • You may want to take the Chapter 10 Practice Test on page 567 of your textbook.

☐ I asked for help from someone else to complete the review of all or most lessons.
  • You should review the examples and concepts in your Study Notebook and Chapter 10 Foldable.
  • Then complete the Chapter 10 Study Guide and Review on pages 563–566 of your textbook.
  • If you are unsure of any concepts or skills, refer to the specific lesson(s).
  • You may also want to take the Chapter 10 Practice Test on page 567 of your textbook.

Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 10.

Student Signature  Parent/Guardian Signature

Teacher Signature
Measurement: Two- and Three-Dimensional Figures

Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

**NOTE-TAKING TIP:** When you take notes, it is helpful to write key vocabulary words, definitions, concepts, or procedures as clearly and concisely as possible.

**STEP 1** Fold the construction paper in half lengthwise. Label the chapter title on the outside.

**STEP 2** Fold the sheets of notebook paper in half lengthwise. Then fold top to bottom twice.

**STEP 3** Open the notebook paper. Cut along the second folds to make four tabs.

**STEP 4** Glue the uncut notebook paper side by side onto the construction paper. Label each tab as shown.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 11. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>base</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>circle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>circumference</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>composite figure</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cone</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cylinder</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>diameter</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>edge</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>face</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Vocabulary Term

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>height</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>lateral face</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>prism</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pyramid</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>radius</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rectangular prism</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>solid</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>sphere</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>three-dimensional figure</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>triangular prism</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>vertex</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>volume</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The base is any side of a parallelogram.

The height is the length of the segment perpendicular to the base with endpoints on opposite sides.

**EXAMPLE** Find the Area of a Parallelogram

Find the area of the parallelogram.

![Parallelogram diagram]

**Estimate** \( A = 8 \cdot 6 \) or \( 48 \) cm\(^2\)

\[ A = bh \]

Area of a parallelogram

\[ A = 7.5 \cdot 6.4 \]

Replace \( b \) with 7.5 and \( h \) with 6.4.

\[ A = 48 \]

Multiply.

The area of the parallelogram is \( 48 \) square centimeters.

This is the same as the estimate.

**Check Your Progress** Find the area of the parallelogram.

![Parallelogram diagram]

52 in\(^2\)
**EXAMPLE**  Find the Area of a Triangle

1. Find the area of the triangle below.

```
\[ \text{Area of a triangle.} \]
```

```
A = \frac{1}{2} bh
```

```
A = \frac{1}{2} (9) (3.2)
```

```
A = 14.4
```

The area of the triangle is 14.4 square centimeters.

This is close to the estimate.

**Check Your Progress**  Find the area of the triangle below.

```
\[ 13.5 \text{ ft}^2 \]
```

**EXAMPLE**  Find the Area of a Trapezoid

1. Find the area of the trapezoid below.

```
The bases are 4 meters and 7.6 meters.
```

```
The height is 3 meters.
```
**Key Concept**

Area of a Trapezoid The area $A$ of a trapezoid equals half the product of the height $h$ and the sum of the bases $b_1$ and $b_2$.

$$A = \frac{1}{2} h(b_1 + b_2)$$

Area of a trapezoid

$$A = \frac{1}{2} (3)(4 + 7.6)$$

Replace $h$ with 3, $b_1$ with 4, and $b_2$ with 7.6.

$$A = \frac{1}{2} (3)(11.6)$$

Add 4 and 7.6.

$$A = 17.4$$

Multiply.

The area of the trapezoid is 17.4 square meters.

**Check Your Progress**

Find the area of the trapezoid below.

![Trapezoid diagram](8 cm, 6 cm, 12.5 cm)

$$61.5 \text{ cm}^2$$
A circle is a set of all points in a plane that are the same distance from a given point called the center.

The diameter \((d)\) is the distance across a circle through its center.

The circumference \((C)\) is the distance around a circle.

The radius \((r)\) is the distance from the center to any point on a circle.

An approximation often used for \(\pi\) (pi) is \(3.14\).

**Example**

Find Circumference

**PETS** Find the circumference around the hamster’s running wheel shown. Round to the nearest tenth.

\[C = 2\pi r\]

\[C = 2(3.14)(3)\]

\[C = 18.8\]

Multiply.

The circumference is about 18.8 inches.
**SWIMMING POOL**

A new children’s swimming pool is being built at the local recreation center. The pool is circular in shape with a diameter of 18 feet. Find the circumference of the pool. Round to the nearest tenth.

\[ C = \pi d \]

Replace \( \pi \) with \( \frac{22}{7} \) and \( d \) with 18.

\[ C \approx \frac{22}{7} \cdot 18 \]

Divide by the GCF, 2.

\[ C \approx 56.5 \]

The circumference is about 56.5 feet.

**Check Your Progress**

Find the circumference of a circle with a radius of 35 feet.

\[ C = \pi r \]

Replace \( \pi \) with \( \frac{22}{7} \) and \( r \) with 35.

\[ C \approx \frac{22}{7} \cdot 35 \]

Multiply.

\[ C \approx 110 \]

The circumference is about 110 feet.

---

**HOMEWORK ASSIGNMENT**

Page(s): 11-3

Exercises: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60
EXAMPLES
Find the Areas of Circles

1. Find the area of the circle at the right.

\[ A = \pi r^2 \quad \text{Area of a circle} \]

\[ A = \pi \cdot 4^2 \quad \text{Replace } r \text{ with } 4. \]

\[ \pi \times 2 \times 4^2 \text{ ENTER } 50.26548246 \]

The area of the circle is approximately 50.3 square centimeters.

2. KOI Find the area of the koi pond shown.

The diameter of the pond is 3.6 meters, so the radius is \( \frac{1}{2}(3.6) \) or 1.8 meters.

\[ A = \pi r^2 \quad \text{Area of a circle} \]

\[ A = \pi \left( \frac{3.6}{2} \right)^2 \quad \text{Replace } r \text{ with } 1.8. \]

\[ A \approx 10.2 \quad \text{Use a calculator.} \]

The area is approximately 10.2 square meters.

Check Your Progress

a. Find the area of the circle below.

b. COINS Find the area of a nickel with a diameter of 2.1 centimeters.
A sector of a circle is a region of a circle bounded by two radii.

**EXAMPLE**

**TEST EXAMPLE** Mr. McGowan made an apple pie with a diameter of 10 inches. He cut the pie into 6 equal slices. Find the approximate area of each slice.

A) 3 in\(^2\)  B) 13 in\(^2\)  C) 16 in\(^2\)  D) 52 in\(^2\)

**Read the Item**
You can use the diameter to find the total area of the pie and then divide that result by 6 to find the area of each slice.

**Solve the Item**
Find the area of the whole pie.

\[ A = \pi r^2 \]  
Area of a circle  
\[ A = \pi \left(\frac{10}{2}\right)^2 \]  
Replace \( r \) with \( 5 \).  
\[ A \approx 78 \]  
Multiply.

Find the area of one slice.

\[ \frac{78}{6} = 13 \]

The area of each slice is approximately 13 square inches.

The correct answer is B.

**Check Your Progress**

**MULTIPLE CHOICE** The floor of a merry-go-round at the amusement park has a diameter of 40 feet. The floor is divided evenly into eight sections, each having a different color. Find the area of each section of the floor.

F) 15.7 ft\(^2\)  H) 62.8 ft\(^2\)  
G) 20 ft\(^2\)  J) 157 ft\(^2\)
EXAMPLE  Use the Solve a Simpler Problem Strategy

PAINT  Ben and Shelia are going to paint the wall of a room as shown in the diagram. What is the area that will be painted?

UNDERSTAND  You know the dimensions of the wall including the door and window. You also know the dimensions of the door and window. You need to find the area of the wall not including the door and window.

PLAN  Find the area of the wall including the door and window. Then subtract the area of the door and the window.

SOLVE  

area of wall including door and window:  
\[ A = lw \]
\[ A = 12 \cdot 9 \text{ or } 108 \text{ square feet} \]

area of door:  
\[ A = lw \]
\[ A = 3 \cdot 7 \text{ or } 21 \text{ square feet} \]

area of window:  
\[ A = lw \]
\[ A = 5 \cdot 4 \text{ or } 20 \text{ square feet} \]

The total area to be painted is 108 - 21 - 20 or 67 square feet.

CHECK  The area to be painted is 67 square feet. Add the area of the door and the window. 67 + 21 + 20 is 108 square feet. So, the answer is correct.

Check Your Progress  Karen is placing a rectangular area rug measuring 8 feet by 10 feet in a rectangular dining room that measures 14 feet by 18 feet. Find the area of the flooring that is not covered by the area rug.

172 ft²
Main Idea
- Find the areas of composite figures.

Build Your Vocabulary (pages 255–256)
A composite figure is made of triangles, quadrilaterals, semicircles, and other two-dimensional figures.
A semicircle is half of a circle.

Example
Find the Area of a Composite Figure
Find the area of the figure in square centimeters.

The figure can be separated into a rectangle and a triangle. Find the area of each.

Area of Rectangle \[ A = \ell w \]
\[ A = 15 \cdot 10 \text{ or } 150 \]

Area of Triangle \[ A = \frac{1}{2}bh \]
\[ A = \frac{1}{2}(5)(4) \text{ or } 10 \]

The area is 150 + 10 or 160 square centimeters.

Check Your Progress
Find the area of the figure shown.

119 yd²
EXAMPLE

**Find the Area of a Composite Figure**

**WINDOWS** The diagram at the right shows the dimensions of a window. Find the area of the window. Round to the nearest tenth.

The figure can be separated into a semicircle and a rectangle.

**Area of Semicircle**

\[ A = \frac{1}{2} \pi r^2 \]

Area of a semicircle

\[ A = \frac{1}{2} \pi (1.7)^2 \]

Replace \( r \) with \( \frac{3.4}{2} \) or 1.7.

\[ A \approx 4.5 \]

Simplify.

**Area of Rectangle**

\[ A = \ell w \]

Area of a rectangle

\[ A = 5.5 \cdot 3.4 \]

Replace \( \ell \) with \( 7.2 - 1.7 \) or 5.5 and \( w \) with 3.4.

\[ A = 18.7 \]

Multiply.

The area of the window is approximately \( 4.5 + 18.7 \) or \( 23.2 \) square feet.

**Check Your Progress** The diagram below shows the dimensions of a new driveway. Find the area of the driveway. Round to the nearest tenth.

**HOMEWORK ASSIGNMENT**

Page(s):

Exercises:

---

Copyright © Glencoe/McGraw-Hill, a division of The McGraw-Hill Companies, Inc.
A three-dimensional figure has length, width, and depth.

A face is a flat surface. The edges are the segments formed by intersecting faces. The edges intersect at the vertices. The sides are called lateral faces.

**Build Your Vocabulary** (pages 255–256)

**Examples**

Classify Three-Dimensional Figures

For each figure, identify the shape of the base(s). Then classify the figure.

1. The figure has four triangular faces and one rectangular base. The figure is a **rectangular pyramid**.

2. The base and all other faces are rectangles. The figure is a **rectangular prism**.

**Check Your Progress**

For each figure, identify the shape of the base(s). Then classify the figure.

a. **Circular; cylinder**

b. **Triangular; triangular pyramid**

**Foldables**

Organize It

Record notes about classifying three-dimensional figures under the tab for Lesson 11-7 of your Foldable.
The top and bottom faces of a three-dimensional figure are called the bases.

A prism has at least three lateral faces that are rectangles.

A pyramid has at least three lateral faces that are triangles.

A cone has one base that is a circle and one vertex.

A cylinder has two bases that are congruent circles.

All of the points on a sphere are the same distance from the center.

**EXAMPLE**

**HOUSES** Classify the shape of the house’s roof as a three-dimensional figure.

The shape of the house’s roof is a triangular prism.

**Check Your Progress** Classify the shape of the house above, not including the roof.

rectangular prism
**EXAMPLE**

**Draw a Three-Dimensional Figure**

Draw a top, a side, and a front view of the figure below.

The top and front views are **rectangles**. The side view is a **square**.

---

**Check Your Progress**

Draw a top, a side, and a front view of the figure below.
**EXAMPLE**

**Draw a Three-Dimensional Figure**
(parallels Example 3 in text)

Draw the three-dimensional figure whose top, side, and front views are shown below. Use isometric dot paper.

![Top, Side, and Front Views](image)

**Step 1** Use the top view to draw the base of the figure.

**Step 2** Add edges to make the base a solid figure.

**Step 3** Use the side and front views to complete the figure.

**Check Your Progress** Draw a solid using the top, side, and front views shown below. Use isometric dot paper.

![Top, Side, and Front Views](image)

---

**Homework Assignment**

Page(s):

Exercises:
**Main Idea**

- Find the volumes of rectangular and triangular prisms.

**Build Your Vocabulary** (pages 255–256)

A volume of a three-dimensional figure is the measure of space occupied by it.

A **rectangular prism** is a prism that has rectangular bases. A triangular prism has **triangular** bases.

**Example** Volume of a Rectangular Prism

Find the volume of the rectangular prism.

\[ V = \ell \cdot w \cdot h \]

The volume is 24 cubic centimeters.

**Check Your Progress**

Find the volume of the rectangular prism.

240 in\(^3\)
EXAMPLE  Find the Volume of a Cylinder

Find the volume of the cylinder. Round to the nearest tenth.

\[ V = \pi r^2 h \]  
Volume of a cylinder

\[ V = \pi (5.5)^2 (9) \]  
Replace the variables.

\[ V \approx 854.9 \]  
Use 3.14 for \( \pi \).

The volume is about 854.9 cubic centimeters.

Check Your Progress  Find the volume of the cylinder. Round to the nearest tenth.

\[ 1,639.1 \text{ in}^3 \]
**EXAMPLE**

**COFFEE** How much coffee can the can hold?

![Cylinder diagram]

\[ V = \pi r^2 h \]

Volume of a cylinder

\[ V = \pi \left( \frac{1.5}{2} \right)^2 \times 6 \]

Replace \( r \) with 1.5 and \( h \) with 6.

\[ V \approx 42.4 \]

Simplify.

The coffee can holds about 42.4 cubic inches.

**WRITE IT**

Explain how you would use a calculator to evaluate a power.

---

**Check Your Progress**

**JUICE** Find the volume of a cylinder-shaped juice can that has a diameter of 5 inches and a height of 8 inches.

\[ V = \pi \left( \frac{5}{2} \right)^2 \times 8 \]

\[ V \approx 157 \text{ in}^3 \]
11-1
Area of Parallelograms

State whether each sentence is true or false. If false, replace the underlined word to make a true sentence.

1. To find the base of a parallelogram, draw a segment perpendicular to the base with endpoints on opposite sides of the parallelogram.  
   false; height

2. The area of a parallelogram is found by multiplying its base times the height.  
   true

3. What is the area of a parallelogram with a base of 15 feet and a height of 3.5 feet?  
   52.5 ft²

11-2
Area of Triangles and Trapezoids

Complete the sentence.

4. To find the height of a triangle, find the distance from the base to the opposite vertex.

Find the area.

5.  
   
   13
   5
   12
   30

6.  
   
   9 in.
   7 in.
   13 in.
   77 in²
11-3 Circles and Circumference

Find the circumference of each circle. Use 3.14 or \( \frac{22}{7} \) for \( \pi \). Round to the nearest tenth if necessary.

7. radius = 7.4 cm
   - 46.5 cm

8. radius = \( 3\frac{1}{2} \) in.
   - 22 in.

9. diameter = \( 6\frac{1}{8} \) ft
   - 19\( \frac{1}{4} \) ft

10. diameter = 1.7 mi
    - 5.3 mi

11-4 Area of Circles

Complete each sentence.

11. To find the area of a circle when you are given the diameter, divide the length of the diameter by \( 2 \), square that, and multiply the result by \( \pi \).

12. The units for the area of a circle will always be measured in square units.

13. Find the area of a circle with a diameter of 13.6 inches. Round to the nearest tenth. \( 145.3 \text{ in}^2 \)

11-5 Problem-Solving Investigation: Solve a Simpler Problem

14. MOVIES Five friends, Marcy, Luke, Shawnda, Jorge, and Lily sat in a row at the movie theater. Marcy and Luke sat next to each other, Jorge did not sit next to Luke, and Shawnda sat at the right end. If Lily sat next to Shawnda and Jorge, find the order of the friends’ seating from left to right.

   Luke, Marcy, Jorge, Lily, Shawnda
Chapter 11 BRINGING IT ALL TOGETHER

11-6 Area of Composite Figures

Name the two dimensions of the following figures.

15. rectangle length and width
16. triangle base and height

Find the area of each figure. Round to the nearest tenth if necessary.

17. 4 in. 8 in. 12 in. 4 in.
   64 in²

18. 3 cm 7 cm
   49.3 cm²

11-7 Three-Dimensional Figures

For each figure, identify the shape of the base(s). Then classify the figure.

19. pentagon; pyramid
20. circle; cylinder

21. MONUMENTS Ginger made a scale model of the Washington Monument as shown. What geometric figure is represented by the top figure of the monument?
   pyramid
Chapter 11 BRINGING IT ALL TOGETHER

11-8 Drawing Three-Dimensional Figures

Complete each sentence.
22. A two-dimensional figure has two dimensions: length and width.
23. A three-dimensional figure has three dimensions: length, width, and depth.

11-9 Volume of Prisms

Find the volume of rectangular prisms with these dimensions. Round to the nearest tenth if necessary.
24. 4 ft by 12 ft by 7 ft
25. 9 in. by 8 in. by 5.5 in.
336 ft$^3$ 396 in$^3$
26. 2.5 in. by 6 in. by 5 in.
27. 3.8 cm by 2.4 cm by 2 cm
75 in$^3$ 18.2 cm$^3$

11-10 Volume of Cylinders

Write C if the phrase is true of a cylinder, P if it is true of a prism, and CP if the phrase is true of both.
28. CP has bases that are parallel and congruent.
29. P has sides and bases that are polygons.
30. C has bases that are circular.
31. CP is a solid.
32. CP has volume.
33. CP is three-dimensional.
ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
  • You are probably ready for the Chapter Test.
  • You may want to take the Chapter 11 Practice Test on page 631 of your textbook as a final check.

☐ I used my Foldable or Study Notebook to complete the review of all or most lessons.
  • You should complete the Chapter 11 Study Guide and Review on pages 626–630 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may want to take the Chapter 11 Practice Test on page 631 of your textbook.

☐ I asked for help from someone else to complete the review of all or most lessons.
  • You should review the examples and concepts in your Study Notebook and Chapter 11 Foldable.
  • Then complete the Chapter 11 Study Guide and Review on pages 626–630 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 11 Practice Test on page 631 of your textbook.

Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 11.

Student Signature  Parent/Guardian Signature  Teacher Signature
Geometry and Measurement

Use the instructions below to make a Foldable to help you organize your notes as you study the chapter. You will see Foldable reminders in the margin of this Interactive Study Notebook to help you in taking notes.

Begin with a sheet of 11" by 17" paper.

**STEP 1** Fold the paper in fourths lengthwise.

**STEP 2** Fold a 2" tab along the short side. Then fold the rest in half.

**STEP 3** Draw lines along folds and label as shown.

<table>
<thead>
<tr>
<th>Ch. 14</th>
<th>Rectangular Prisms</th>
<th>Cylinders</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**NOTE-TAKING TIP:** When taking notes about 3-dimensional figures, it is important to draw examples. It also helps to record any measurement formulas.
This is an alphabetical list of new vocabulary terms you will learn in Chapter 12. As you complete the study notes for the chapter, you will see Build Your Vocabulary reminders to complete each term’s definition or description on these pages. Remember to add the textbook page number in the second column for reference when you study.

<table>
<thead>
<tr>
<th>Vocabulary Term</th>
<th>Found on Page</th>
<th>Definition</th>
<th>Description or Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>hypotenuse</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>irrational number</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>leg</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pythagorean Theorem</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>surface area</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
EXAMPLE 1 Estimate the Square Root

Estimate \( \sqrt{96} \) to the nearest whole number.

List some perfect squares.

\[ 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, \ldots \]

\[ 81 < 96 < 100 \]

\( 96 \) is between the perfect squares \( 81 \) and \( 100 \).

\[ \sqrt{81} < \sqrt{96} < \sqrt{100} \]

Find the \( \sqrt{} \) of each number.

\[ \sqrt{81} = 9 \]

\[ \sqrt{100} = 10 \]

So, \( \sqrt{96} \) is between \( 9 \) and \( 10 \). Since \( 96 \) is closer to \( 100 \) than \( 81 \), the best whole number estimate is \( 10 \). Verify with a calculator.

Check Your Progress Estimate each square root to the nearest whole number.

a. \( \sqrt{41} \)

\[ 6 \]

b. \( \sqrt{86} \)

\[ 9 \]

c. \( \sqrt{138} \)

\[ 12 \]
**BUILD YOUR VOCABULARY** (page 280)

A number that cannot be written as a fraction is an irrational number.

**EXAMPLE** Use a Calculator to Estimate

Graph $\sqrt{37}$ on a number line.

1. Graph $\sqrt{37}$ on a number line.
   
   $\sqrt{37} \approx 6.1$

   Check $6^2 = 36$ and $7^2 = 49$. Since $37$ is between 36 and 49, the answer, $6.1$, is reasonable.

**Check Your Progress** Graph each number on a number line.

a. $\sqrt{78}$
   
   $\sqrt{78} \approx 8.8$

b. $\sqrt{96}$
   
   $\sqrt{96} \approx 9.8$

c. $\sqrt{188}$
   
   $\sqrt{188} \approx 13.7$
The two sides adjacent to the right angle of a right triangle are the legs.

The side opposite the right angle of a right triangle is the hypotenuse.

The Pythagorean Theorem describes the relationship between the length of the hypotenuse and the lengths of the legs.

**EXAMPLE** Find the Length of the Hypotenuse

Find the length of the hypotenuse of the triangle.

\[c^2 = a^2 + b^2\]  \hspace{1cm} \text{Pythagorean Theorem}

\[c^2 = 2^2 + 6^2\]  \hspace{1cm} \text{Replace } a \text{ with } 2 \text{ and } b \text{ with } 6.

\[c^2 = 4 + 36\]  \hspace{1cm} \text{Evaluate } 2^2 \text{ and } 6^2.

\[c^2 = 40\]  \hspace{1cm} \text{Add.}

\[c = \pm\sqrt{40}\]  \hspace{1cm} \text{Definition of square root}

\[c = \pm6.3\]  \hspace{1cm} \text{Simplify.}

The length of the hypotenuse is about 6.3 millimeters.
Check Your Progress Find the length of the hypotenuse of a right triangle if the legs are 5 centimeters and 7 centimeters.

8.6 cm

EXAMPLE SPORTS A gymnastics tumbling floor is in the shape of a square. If a gymnast flips from one corner to the opposite corner, about how far has he flipped?

To solve, find the length of the hypotenuse $c$.

$$c^2 = a^2 + b^2$$ Pythagorean Theorem

$$c^2 = 12^2 + 12^2$$ Replace $a$ with 12 and $b$ with 12.

$$c^2 = 144 + 144$$ Evaluate $12^2$.

$$c^2 = 288$$ Add.

$$\sqrt{c^2} = \pm \sqrt{288}$$ Take the square root of each side.

$$c \approx \pm 17.0$$ Simplify.

The gymnast will have flipped about 17 meters.

Check Your Progress SEWING Rose has a rectangular piece of fabric 28 inches long and 16 wide. She wants to decorate the fabric with lace sewn across both diagonals. How much lace will Rose need?

about 64.5 in.
EXAMPLE

Find the Length of a Leg

Find the missing measure of the triangle at the right.

\[ c^2 = a^2 + b^2 \]

\[ 15^2 = a^2 + 9^2 \]

Pythagorean Theorem

Replace \( b \) with 9 and \( c \) with 15.

\[ 225 = a^2 + 81 \]

Evaluate \( 15^2 \) and \( 9^2 \).

Subtract 81 from each side.

\[ 144 = a^2 \]

Simplify.

\[ \sqrt{144} = \sqrt{a^2} \]

Take the square root of each side.

\[ 12 = a \]

Simplify.

The length of the leg is 12 centimeters.

Check Your Progress

Find the missing measure of the triangle. Round to the nearest tenth if necessary.

\[ 7 \text{ in.} \]

\[ 20 \text{ in.} \]

b in.

18.7 in.
EXAMPLE  Make a Model to Solve the Problem

**STORAGE** A daycare center plans to make simple wooden storage bins for the 3-inch square alphabet blocks. If each bin will hold 30 blocks, give two possible dimensions for the inside of the bin.

**UNDERSTAND** You know the dimensions of the blocks and that each bin holds 30 blocks. You need to give two possible dimensions for the inside of the bin.

**PLAN** Make a cardboard model of a cube with sides 3 inches long. Then use your model to determine the dimensions of the bin that will hold 30 cubes.

**SOLVE**

A bin that holds 5 cubes in length, 3 cubes in width, and 2 cubes in height would hold 30 cubes. This bin would be 15 inches in length, 9 inches in width, and 6 inches in height. A bin that holds 6 cubes in length, 5 cubes in width, and 1 cube in height would also hold 30 cubes. This bin would be 18 inches in length, 15 inches in width, and 3 inches in height.

**CHECK**

A bin that is 15 in. \(\times\) 9 in. \(\times\) 6 in. would hold

\[15 \div 3 \text{ or } 5\text{ cubes by } 9 \div 3 \text{ or } 3\text{ cubes by } 6 \div 3 \text{ or } 2\text{ cubes in height.} \]

This is \(5 \times 3 \times 2\text{ or } 30\text{ cubes.} \)

A bin that is 18 in. \(\times\) 15 in. \(\times\) 3 in. would hold

\[18 \div 3 \text{ or } 6\text{ cubes by } 15 \div 3 \text{ or } 5\text{ cubes by } 3 \div 3 \text{ or } 1\text{ cube. This is } 6 \times 5 \times 1\text{ or } 30\text{ cubes.} \]

**Check Your Progress**  FRAMES A photo that is 5 inches by 7 inches will be placed in a frame that has a metal border of 1.5 inches on each side. What are the dimensions of the frame?

8 in. by 10 in.
**Main Idea**

- Find the surface areas of rectangular prisms.

**Build Your Vocabulary**

The **sum** of the areas of all of the **surfaces** of faces, of a **three-dimensional** figure is the **surface area**.

**Example**

**Use a Net to Find Surface Area**

**Find the Surface Area of the Rectangular Prism.**

You can use a net of the rectangular prism to find its surface area. There are three pairs of congruent faces.

- **top and bottom**
- **front and back**
- **two sides**

<table>
<thead>
<tr>
<th>Faces</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>top and bottom</td>
<td>$2(6 \cdot 2) = 24$</td>
</tr>
<tr>
<td>front and back</td>
<td>$2(6 \cdot 3) = 36$</td>
</tr>
<tr>
<td>two sides</td>
<td>$2(2 \cdot 3) = 12$</td>
</tr>
</tbody>
</table>

The surface area is $24 + 36 + 12$ or $72$ square centimeters.

**Check Your Progress**

Find the surface area of the rectangular prism.

$150 \text{ ft}^2$
**EXAMPLE**

**Use a Formula to Find Surface Area**

Find the surface area of the rectangular prism.

Replace \( \ell \) with 10, \( w \) with 8, and \( h \) with 12.

\[
\text{surface area} = 2\ell w + 2\ell h + 2wh
\]

\[
= 2 \cdot 10 \cdot 8 + 2 \cdot 10 \cdot 12 + 2 \cdot 8 \cdot 12
\]

\[
= 160 + 240 + 192
\]

\[
= 592
\]

The surface area of the prism is \( 592 \text{ square inches} \).

**Check Your Progress**

Find the surface area of the rectangular prism.

Replace \( \ell \) with 10, \( w \) with 8, and \( h \) with 12.

\[
\text{surface area} = 2\ell w + 2\ell h + 2wh
\]

\[
= 2 \cdot 10 \cdot 8 + 2 \cdot 10 \cdot 12 + 2 \cdot 8 \cdot 12
\]

\[
= 160 + 240 + 192
\]

\[
= 592
\]

The surface area of the prism is \( 592 \text{ square inches} \).
EXAMPLE

BOXES Drew is putting together a cardboard box that is 9 inches long, 6 inches wide, and 8 inches high. He bought a roll of wrapping paper that is 1 foot wide and 3 feet long. Did he buy enough to wrap the box? Explain.

Step 1 Find the surface area of the box.

Replace \( \ell \) with 9, \( w \) with 6, and \( h \) with 8.

\[
\text{surface area} = 2 \cdot 9 \cdot 6 + 2 \cdot 9 \cdot 8 + 2 \cdot 6 \cdot 8 = 348 \text{ in}^2
\]

Step 2 Find the area of the wrapping paper.

\[
\text{area} = 12 \text{ in.} \cdot 36 \text{ in. or } 432 \text{ in}^2
\]

Since 432 > 348, Drew bought enough wrapping paper.

Check Your Progress FABRIC Angela needs to cover a cardboard box that is 15 inches long, 5 inches wide, and 4 inches high with felt. She bought a piece of felt that is 1 foot wide and \( 2 \frac{1}{2} \) feet long. Did she buy enough felt to cover the box? Explain.

Yes; the surface area of the box is 310 in\(^2\) and the area of the felt is 360 in\(^2\). Since 360 > 310, Angela bought enough felt.
EXAMPLE  Find Surface Area of a Cylinder

Find the surface area of the cylinder. Round to the nearest tenth.

\[
S = 2\pi r^2 + 2\pi rh
\]

Surface area of a cylinder

\[
= 2\pi (3)^2 + 2\pi (3)(6)
\]

\[
= 169.6
\]

Simplify.

The surface area is about 169.6 square centimeters.

EXAMPLE  GIFT WRAP  A poster is contained in a cardboard cylinder that is 10 inches high. The cylinder’s base has a diameter of 8 inches. How much paper is needed to wrap the cardboard cylinder if the ends are to be left uncovered?

Since only the curved side of the cylinder is to be covered, you do not need to include the areas of the top and bottom of the cylinder.

\[
S = 2\pi rh
\]

Curved surface of a cylinder

\[
= 2\pi (4)(10)
\]

\[
= 251.2
\]

Simplify.

About 251.3 square inches of paper is needed.
Check Your Progress

a. Find the surface area of the cylinder. Round to the nearest tenth.

\[ \text{Surface Area} = \pi r (r + h) \]

\[ r = 8 \text{ in}, \quad h = 15 \text{ in} \]

\[ \text{Surface Area} = \pi (8)(8 + 15) \]

\[ = \pi (8)(23) \]

\[ = 1,156.1 \text{ in}^2 \]

b. LABELS A can of fruit juice is in the shape of a cylinder with a diameter of 6 inches and a height of 12 inches. How much paper is needed to create the label if the ends are to be left uncovered?

\[ \text{Surface Area} = \pi r (r + h) \]

\[ r = 3 \text{ in}, \quad h = 12 \text{ in} \]

\[ \text{Surface Area} = \pi (3)(3 + 12) \]

\[ = \pi (3)(15) \]

\[ = 226.2 \text{ in}^2 \]
12-1  Estimating Square Roots

Estimate each square root to the nearest whole number.

1. \( \sqrt{95} \)  
   10

2. \( \sqrt{51} \)  
   7

3. \( \sqrt{150} \)  
   12

4. \( \sqrt{230} \)  
   15

12-2  The Pythagorean Theorem

State whether each sentence is true or false. If false, replace the underlined word to make a true sentence.

5. The Pythagorean Theorem states that \( c^2 = a^2 + b^2 \), where \( c \) represents the length of the hypotenuse.  
   false; \( c \)

6. The hypotenuse is always the longest of the three sides of a right triangle.  
   true

Find the missing measure of each right triangle. Round to the nearest tenth if necessary.

7.  
   \( \begin{array}{c}
   3 \text{ in.} \\
   8 \text{ in.}
   \end{array} \)  
   8.5 in.

8.  
   \( \begin{array}{c}
   24 \text{ yd} \\
   12 \text{ yd}
   \end{array} \)  
   26.8 yd
Problem-Solving Investigation: Make a Model

9. BOOKS A bookstore will arrange 4 books in a row in the store window. In how many different ways can the store arrange these 4 books?

24

Surface Area of Rectangular Prisms

Find the surface area of each rectangular prism. Round to the nearest tenth if necessary.

10. 324 in²

11. 11.7 ft²

12. 684 cm²
12-5

Surface Area of Cylinders

Write the formula to find each of the following.

13. the area of a circle \[ A = \pi r^2 \]

14. the circumference of a circle \[ C = 2\pi r \]

15. the area of a rectangle \[ A = bh \]

Find the surface area of the cylinder. Round to the nearest tenth if necessary.

16. \[ \text{Surface Area} = 245.0 \text{ in}^2 \]
ARE YOU READY FOR THE CHAPTER TEST?

Check the one that applies. Suggestions to help you study are given with each item.

☐ I completed the review of all or most lessons without using my notes or asking for help.
  • You are probably ready for the Chapter Test.
  • You may want to take the Chapter 12 Practice Test on page 663 of your textbook as a final check.

☐ I used my Foldable or Study Notebook to complete the review of all or most lessons.
  • You should complete the Chapter 12 Study Guide and Review on pages 660–662 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may want to take the Chapter 12 Practice Test on page 663 of your textbook.

☐ I asked for help from someone else to complete the review of all or most lessons.
  • You should review the examples and concepts in your Study Notebook and Chapter 12 Foldable.
  • Then complete the Chapter 12 Study Guide and Review on pages 660–662 of your textbook.
  • If you are unsure of any concepts or skills, refer back to the specific lesson(s).
  • You may also want to take the Chapter 12 Practice Test on page 663 of your textbook.

Visit glencoe.com to access your textbook, more examples, self-check quizzes, and practice tests to help you study the concepts in Chapter 12.

Student Signature  Parent/Guardian Signature  Teacher Signature